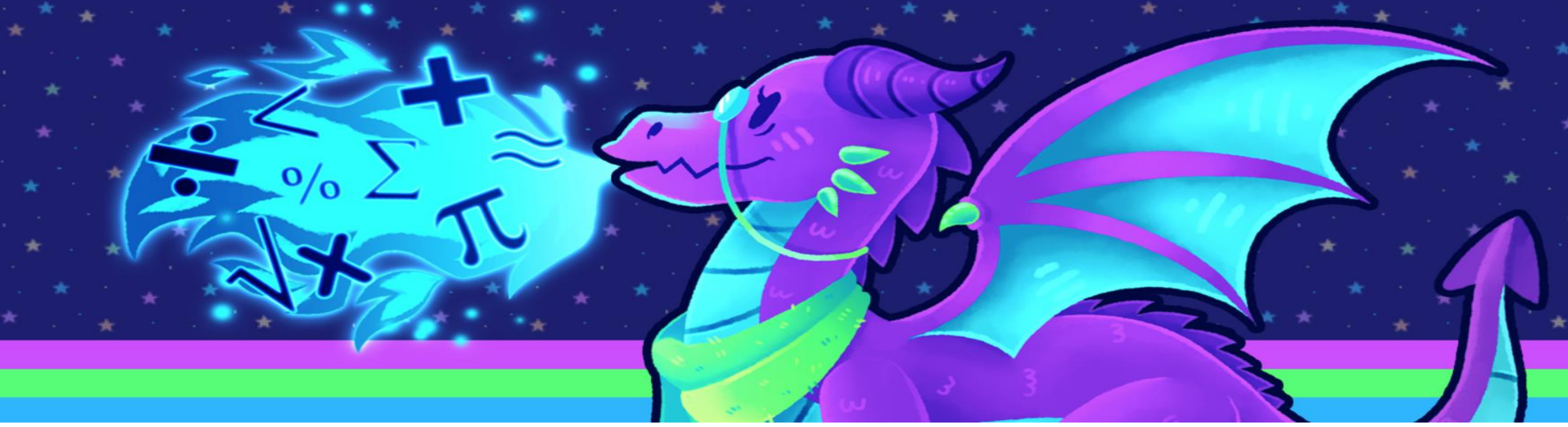

Baldragon Academy

Higher Maths

Checklist

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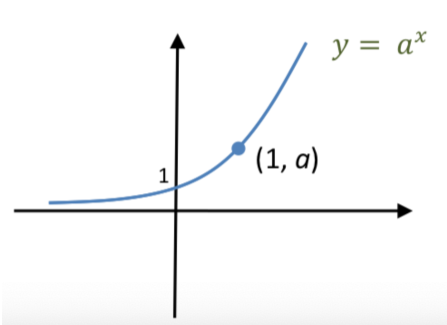
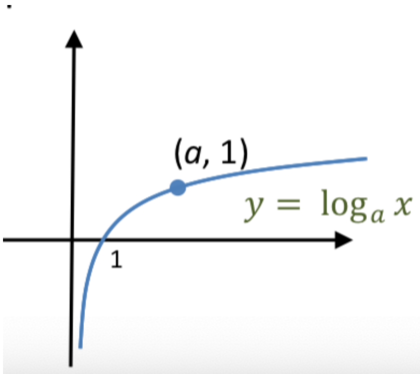
Expressions and Functions

Relationships and Calculus

Applications

Expressions and Functions

Topic	Skills	Notes			
Logs and Exponentials					
Exponential Functions	An exponential function is written in the form: $$y = a^x$$ where a is the base, and x is the exponent  				
The Logarithmic Function	The logarithmic function is the inverse of the exponential function. It is written as: $$y = \log_a x$$ where a is the base  Note: On your calculator the *log* button is $\log_{10} x$				
Convert Between Logarithmic and Exponential Form	If $y = a^x$ then $x = \log_a y$ Example: $3 = \log_a 8$ $a^3 = 8$ $a = \sqrt[3]{8}$ $a = 2$				

The Exponential Function	The exponential function is written as $y = e^x$,where e is the base, which is approximately 2.718				
Natural Logarithms	The natural log function is the inverse of the exponential function $y = e^x$, it is written as $$y = \ln x$$ which means $y = \log_e x$				
Laws of Logs	• $\log_a xy = \log_a x + \log_a y$ Example: $$\log_4 8 + \log_4 2 = \log_4(8 \times 2) = \log_4 16 = 2$$ • $\log_a \frac{x}{y} = \log_a x - \log_a y$ Example: $$\log_4 8 - \log_4 2 = \log_4 \frac{8}{2} = \log_4 4 = 1$$ • $\log_a x^n = n \log_a x$ Example: $$\frac{1}{3}\log_9 27 = \log_9 27^{\frac{1}{3}} = \log_9 3 = \frac{1}{2}$$ • $\log_a a = 1$ Example: $$\log_5 5 = 1$$				
Use the Laws of Logs to Solve Log Equations	Example: Solve: $\log_5(x+1) + \log_5(x-3) = 1$ $$\log_5(x+1)(x-3) = 1 \quad (using\ 1st\ law)$$ $$(x+1)(x-3) = 5$$ $$x^2 - 2x - 3 = 5$$ $$x^2 - 2x - 8 = 0$$ $$(x+2)(x-4) = 0$$ $$\therefore x = -2, \quad x = 4$$ Example:				

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Find x if $4\log_x 6 - 2\log_x 4 = 1$

$$\log_x 6^4 - \log_x 4^2 = 1$$
$$\log_x \frac{6^4}{4^2} = 1$$
$$\frac{6^4}{4^2} = x$$
$$x = \frac{2^4 \times 3^4}{2^4}$$
$$x = 3^4$$
$$x = 81$$

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Use Laws of Logs to Solve Exponential Growth or Decay Problems

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- For an initial value; substitute given values in to equation to determine the initial value.
- For finding a half-life, make the equation equal to one half

Example:

In the equation, where A represents micrograms of a radioactive substance remaining over time t. Find:

(a) The initial value if there are 500 microgram after 100 years.
(b) The half-life of the substance

(a)
$$A_t = A_0 e^{-0.004t}$$
$$500 = A_0 e^{-0.004 \times 100}$$
$$500 = 0.67A_0$$
$$A_0 = 746 \text{ micrograms}$$

(b)
$$373 = 746e^{-0.004t}$$
$$\frac{1}{2} = e^{-0.004t}$$
$$\ln\frac{1}{2} = \ln e^{-0.004t}$$
$$-0.004t = \ln\frac{1}{2}$$
$$t = 173 \text{ years}$$

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Formulae for Experimental Data

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In experimental data questions, two types of exponential functions are considered, $y = kx^n$ and $y = ab^x$

$$y = kx^n$$

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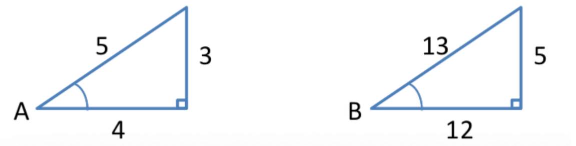
	Taking logs of both sides, this equation may be expressed as $\log y = n \log x + \log k$. To find the unknown values n and k: • If the data given is x and y data, then take logs of two sets of the data for x and y and form a new table with $\log x$ and $\log y$ • Substitute new values into $\log y = n \log x + \log k$ and solve simultaneously to find values for n and $\log k$ • Find k by solving $\log k$ • Write $y = kx^n$ with values of k and n $y = ab^x$ Taking logs of both sides, this equation may be expressed as $\log y = x \log b + \log a$. To find the unknown values a and b: • If the data given is x and y data, then take logs of the data for y. • Substitute values into $\log y = x \log b + \log a$ and solve simultaneously to find values for $\log a$ and $\log b$ • Find a and b by solving $\log a$ and $\log b$ • Write $y = ab^x$ with values of a and b				
Sketch the Graph of the Inverse Function of a Log or Exponential Function	See Graphs of Functions				
Addition Formulae					
Use Exact Values to Calculate Related Obtuse Angles	Example: Find the exact value of $\cos 225°$ The related acute angle is 45° since 180° + 45° = 225° From the graph or CAST diagram cos225 is negative. $\therefore \cos 225° = -\cos 45° = -\frac{1}{\sqrt{2}}$ Example:				

	Find the exact value of $\sin\frac{2\pi}{3}$ The related acute angle is $\frac{\pi}{3}$ since $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$ From the graph or CAST diagram $\sin\frac{2\pi}{3}$ is positive. $\therefore\ \sin\frac{2\pi}{3} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$				
Use Addition Formulae to Expand Expressions	$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ Note: For sin functions the signs are the same, for cos functions the signs are different				
Use Addition Formulae to Evaluate Exact Values of Expressions	Example: Find the exact value of $\cos 75°$ $\cos 75° = \cos(45+30)°$ $= \cos 45°\cos 30° - \sin 45°\sin 30°$ $= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$ $= \frac{\sqrt{3}-1}{2\sqrt{2}}$ Example: Given $\sin A = \frac{3}{5}$ and $\cos B = \frac{12}{13}$, show that $\sin(A+B) = \frac{56}{65}$ Use SOHCAHTOA to sketch triangles from the info given and use Pythagoras to find the unknowns.  Expand using Addition Formulae $\sin(A+B) = \sin A\cos B + \cos A\sin B$				

	$= \frac{3}{5} \times \frac{12}{13} + \frac{4}{5} \times \frac{5}{13}$ $= \frac{56}{65}$				
Solve Trig Equations using Trig Identities	• Determine which part of the equation has a related identity • Replace trigonometric term with related identity and solve Example: Solve $\sin 2x = 2\sin x\ \cos x$, As $\sin 2x = 2\sin x\ \cos x$, then $2\sin x \cos x + \sin x = 0$, solve by factoring Factorise: $\sin x\ (2\cos x + 1) = 0$ $\sin x = 0$ $\qquad$ $2\cos x + 1 = 0$ Using Graph: $\qquad$ $cosx = -\frac{1}{2}$ $x = 0°, 180°$ $\qquad$ Using Cast: Q1 = 60° Q2 = 180° – 60° = 120°				
Wave Function					
Write and Expression Form $k\sin(x \pm \alpha)$ or $k\cos(x \pm \alpha)$	$a\cos x + b\ \sin x$ can be written in one of the following forms: $k\sin(x + \alpha)$ $k\sin(x - \alpha)$ $k\cos(x + \alpha)$ $k\cos(x - \alpha)$ Where $k = \sqrt{a^2 + b^2}$ and $\tan\alpha$ is derived from a and b Example: $k\sin(x + \alpha) = k(\sin x\cos\alpha + \cos x\sin\alpha)$ $= k\cos\alpha\sin x + k\sin\alpha\cos x$ $= \sqrt{3}\ \sin x + \cos x$ $\therefore k\cos\alpha = \sqrt{3}$ and $k\sin\alpha = 1$				

To find k: $k = \sqrt{3+1} = 2$

To find α: $\tan\alpha = \frac{k\sin\alpha}{k\cos\alpha}$

$$\tan\alpha = \frac{1}{\sqrt{3}} \qquad \text{(use exact values)}$$
$$\alpha = 30°$$

Note: $k\sin\alpha$ and $k\cos\alpha$ are both positive, therefore the angle is in Q1, less than 90°

$\therefore\ \sqrt{3}\ \sin x + \ \cos x = 2\ \sin(x+30)°$

Example:

Express $8\cos x - 6\sin x$ in the form $k\cos(x+\ \alpha)°$ where $k\ > 0$ and $0\ \leq x\ \leq 360°$

$$\begin{aligned} k\cos(x+\ \alpha) &= k(\cos x\cos\alpha -\ \sin x\sin\alpha) \\ &= k\cos\alpha\cos x - k\sin\alpha\sin x \\ &= 8\cos x - 6\sin x \end{aligned}$$

$\therefore k\cos\alpha = 8$ and $k\sin\alpha = 6$

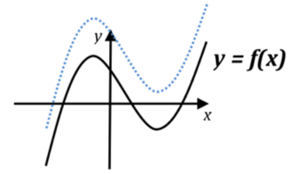
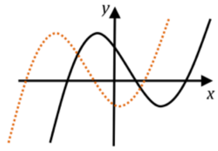
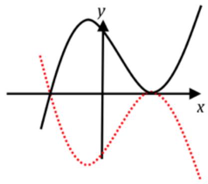
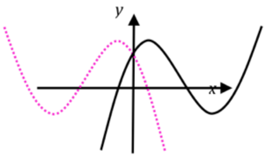
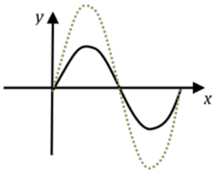
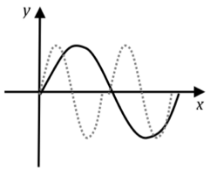
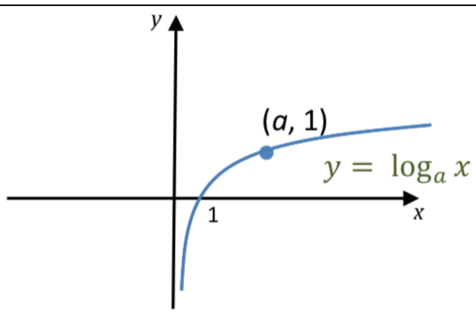
To find k: $k = \sqrt{8^2 +\ 6^2} = 10$

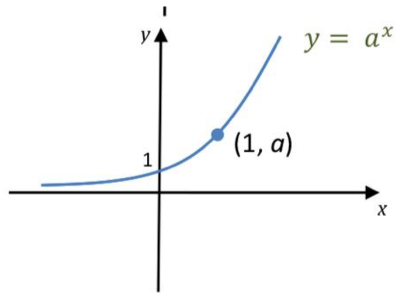
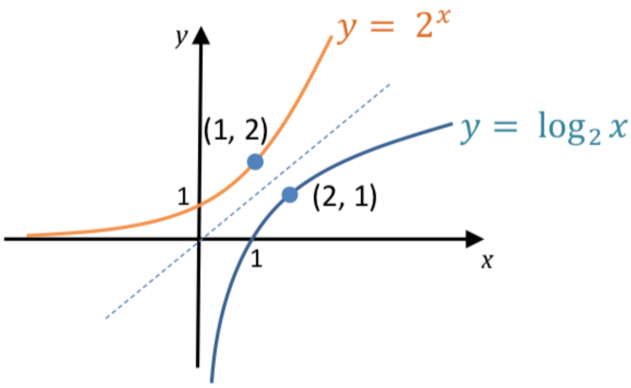
To find α: $\tan\alpha = \frac{k\ \sin\alpha}{k\ \cos\alpha}$

$$\tan\alpha = \frac{6}{8}$$
$$\alpha = 36.9°$$

$\therefore\quad 8\cos x - 6\sin x = 10\cos(x+36.9)°$

Graphs of Functions

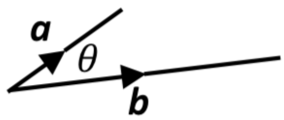
Sketch Related Graphs	Ensure all given coordinates are translated and marked on the new graph and axes and graphs are labelled $y = f(x) + a$ Graph moves up or down by a *Up* for $f(x) + a$ *Down* for $f(x) - a$  $y = f(x + a)$ Graph moves left or right *Left* when $f(x + a)$ *Right* for $f(x - a)$  $y = -f(x)$ Graph reflects in *x*-axis  $y = f(-x)$ Graph reflects in *y*-axis  $y = kf(x)$ Graph is stretched vertically for $k > 1$ Graph is squashed vertically for $0 < k < 1$  $y = f(kx)$ Graph is squashed horizontally for $k > 1$ Graph is stretched horizontally for $0 < k < 1$  				
Sketch Log and Exponential Graphs	Log Graphs of the form $y = \log_a x$ always cuts the x-axis at the point (1,0) and will pass through (a, 1)  				

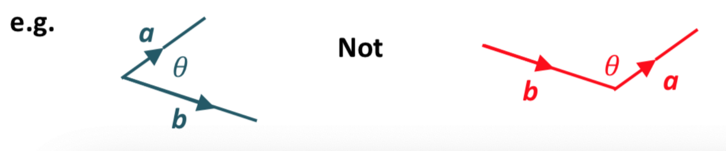
	Exponential graphs of the form $y = a^x$ always cuts the y-axis at the point (0,1) and will pass through (1, a)  All of the related graph transformations above apply to log and exponential graphs.				
Sketch the Graph of the Inverse Function of a Log or Exponential Function	The graph of an inverse function is reflected along the line y=x. the logarithmic graph is the inverse of the exponential graph and vice-versa. Example: For the graph of the function $y = 2^x$ the inverse function is $y = \log_2 x$  				
Sketch a Trig Graph of the Form $k\sin(x \pm \alpha)$ or $k\cos(x \pm \alpha)$	See Trigonometry – Sketch a Trig Graph from its Equation.				
Sets of Functions					
Find Composite Functions	Composite functions consist of one function within another. Example: If $f(x) = 3x - 2$ and $g(x) = x^2 - 4$, find (a) $f(g(x))$ (b) $g(f(x))$				

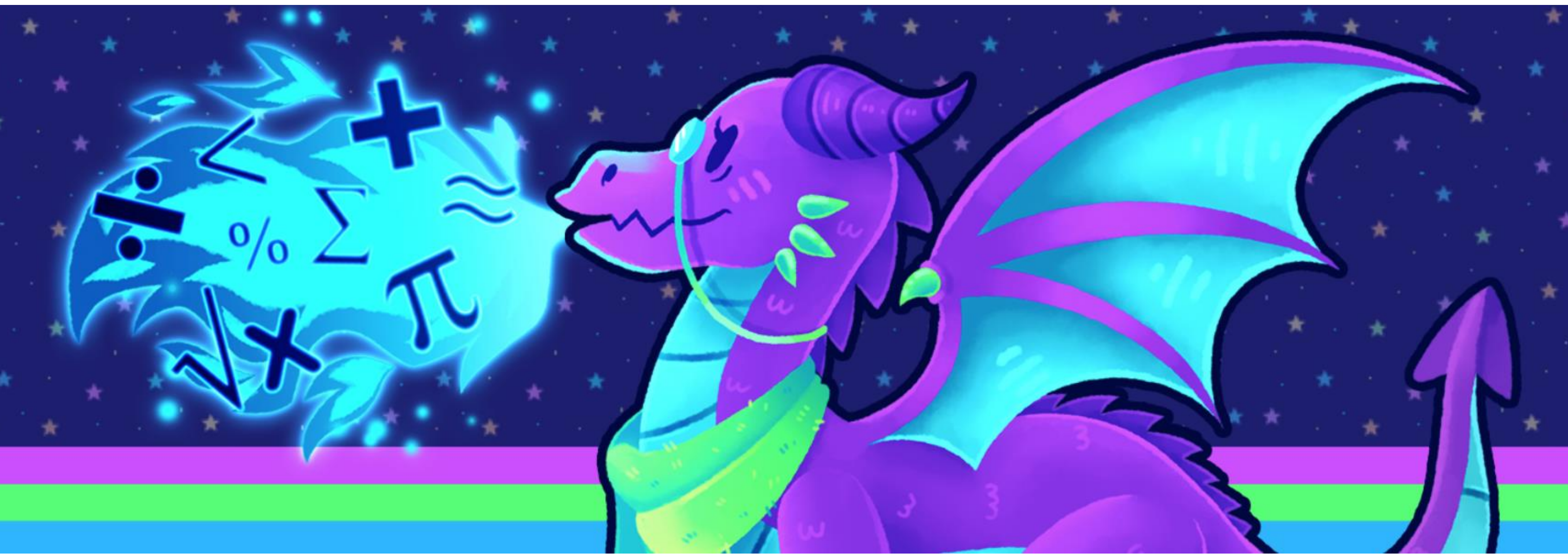
	(a) $f(g(x)) = 3(x^2 - 4) - 2 = 3x^2 - 14$ (b) $g(f(x)) = (3x - 2)^2 - 4 = 9x^2 - 12x$				
Evaluate Using Composite Functions	Example: Find H(-1) where $H(x) = g(f(x))$ and $f(x) = 3x - 2$, $g(x) = x^2 - 4$ Method 1: $H(x) = g(f(x)) = \cdots = 9x^2 - 12x$ $H(-1) = 9(-1)^2 - 12(-1) = 21$ Method 2: $f(-1) = 3(-1) - 2 = -5$ $g(-5) = (-5)^2 - 4 = 21$				
Determine a Suitable Domain of a Function	Restrictions on the domain of a function occur in 2 instances. A restriction will occur when a denominator is zero, which is undefined, or when a square root is negative, which is non-real. Example: For $f(x) = \frac{12x}{(4-x)^2}$ and $x \in \mathbb{R}$, write a restriction on the domain of $f(x)$ $x \neq 4$ as this would make the denominator zero				
State the Range of a Function	Example: State the minimum turning point of the function $f(x) = x^2 + 5$ and hence state the range of the function Minimum turning point is (0, 5) as the y-coordinate of the turning point is 5, the range of the function is $f(x) > 5$.				
Find an Inverse Function	For a function $f(x)$ there is an inverse function $f^{-1}(x)$, such that $f(f^{-1}(x)) = x$ To find an inverse function:				

	• Replace x with y in the function and f(x) with x • Change the subject to y Example: For the function $f(x) = \frac{3}{4 - x^2}$ find the inverse function $f^{-1}(x)$ $f(x) = \frac{3}{4 - x^2}$ $x = \frac{3}{4 - y^2}$ $4 - y^2 = \frac{3}{x}$ $4 - \frac{3}{x} = y^2$ $y = \sqrt{4 - \frac{3}{x}}$ $\therefore\ f^{-1}(x) = \sqrt{4 - \frac{3}{x}}$				
Vectors					
Writing Vectors	Vectors can be written in component form i.e. $\boldsymbol{a} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ or in terms of **i**, **j**, and **k**, where each of these represents the unit vector in the x, y, and z direction. Example: $\overrightarrow{AB} = 4\boldsymbol{i} - 3\boldsymbol{j} + 6\boldsymbol{k}$ can be written as $\overrightarrow{AB} = \begin{pmatrix} 4 \\ -3 \\ 6 \end{pmatrix}$				
Parallel Vectors	Vectors are parallel if one vector is a scalar multiple of the other Example: $\boldsymbol{a} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 4 \\ 4 \\ 16 \end{pmatrix} = 4\begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$ $\boldsymbol{b} = 4\boldsymbol{a}\ \therefore\ vectors\ are\ parallel$				

The Unit Vector	For any vector, there is a parallel vector **u** of magnitude 1. This is called the unit vector Example: Find the unit vector **u** parallel to vector **a** = $\begin{pmatrix} 5 \\ 12 \end{pmatrix}$ $\lvert\mathbf{a}\rvert = \sqrt{5^2 + 12^2} = 13$ $$\therefore \quad \boldsymbol{u} = \frac{1}{13}\begin{pmatrix} 5 \\ 12 \end{pmatrix} = \begin{pmatrix} \frac{5}{13} \\ \frac{12}{13} \end{pmatrix}$$				
Collinearity	Points are said to be collinear is they lie on the same line. To show points are collinear using vectors; show (a) they are parallel by demonstrating one vector is a scalar multiple of the other and (b) that they share a common point. Example: Show that A(-3, 4, 7), B(-1, 8, 3) and C(0, 10, 1) are collinear $$\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a} = \begin{pmatrix} -1 \\ 8 \\ 3 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$$ $$\overrightarrow{BC} = \boldsymbol{c} - \boldsymbol{b} = \begin{pmatrix} 0 \\ 10 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ 8 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$$ $$\overrightarrow{AB} = 2\,\overrightarrow{BC} \text{ and point } B \text{ is common}$$ $$\therefore A, B, \text{and } C \text{ are collinear}$$				
Divide Vectors into a Given Ratio to Find an Unknown Point	Example: P is the point (6, 3, 9) and R is (12, 6, 0). Find the coordinates of Q, such that Q divides PR in the ratio 2:1 $$\frac{\overrightarrow{PQ}}{\overrightarrow{QR}} = \frac{2}{1}$$ $$\overrightarrow{PQ} = 2\,\overrightarrow{QR}$$ $$\boldsymbol{q} - \boldsymbol{p} = 2(\boldsymbol{r} - \boldsymbol{q})$$ $$\boldsymbol{q} - \boldsymbol{p} = 2\boldsymbol{r} - 2\boldsymbol{q}$$				

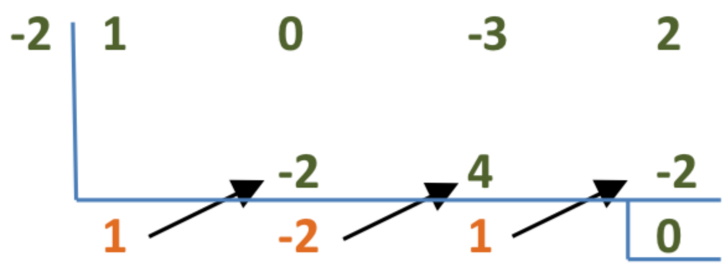
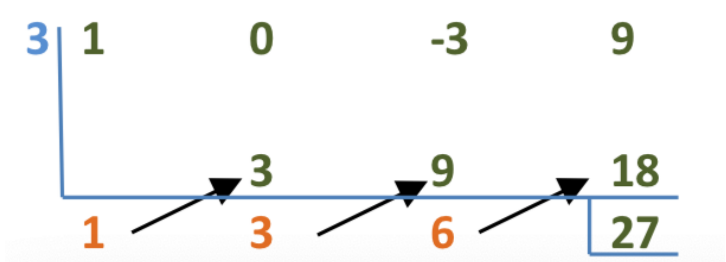
	$3\boldsymbol{q} = 2\boldsymbol{r} + \boldsymbol{p}$ $3\boldsymbol{q} = 2\begin{pmatrix}12\\6\\0\end{pmatrix} + \begin{pmatrix}6\\3\\9\end{pmatrix} = \begin{pmatrix}30\\15\\9\end{pmatrix}$ $\boldsymbol{q} = \begin{pmatrix}10\\5\\3\end{pmatrix} \quad \therefore Q(10,5,3)$ Note: This could also be calculated using section formula				
Find the Ratio in Which a Point Divides a Line Segment	Example: A(-2, -1, 4), B(1, 5, 7) and C(7, 17, 13) are collinear. What is the ratio in which B divides AC? $\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a} = \begin{pmatrix}1\\5\\7\end{pmatrix} - \begin{pmatrix}-2\\-1\\4\end{pmatrix} = \begin{pmatrix}3\\6\\3\end{pmatrix}$ $\overrightarrow{BC} = \boldsymbol{c} - \boldsymbol{b} = \begin{pmatrix}7\\17\\13\end{pmatrix} - \begin{pmatrix}1\\5\\7\end{pmatrix} = \begin{pmatrix}6\\12\\6\end{pmatrix} = 2\begin{pmatrix}3\\6\\3\end{pmatrix} = 2\,\overrightarrow{AB}$ $2\,\overrightarrow{AB} = \overrightarrow{BC}$ $\frac{\overrightarrow{AB}}{\overrightarrow{BC}} = \frac{1}{2}$ $\therefore \overrightarrow{AB} : \overrightarrow{BC}$ $1 : 2$				
Scalar Product	When given an angle between two vectors, the scalar product is calculated using $\boldsymbol{a}.\boldsymbol{b} = \|\boldsymbol{a}\|\|\boldsymbol{b}\|\cos\theta$  Note: To find the angle between the two other vectors θ, the vectors must be pointing away from each other and $0 \leq \theta \leq 180°$ When given component form, i.e. if $\boldsymbol{a} = \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$, the scalar product is calculated using $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$				
Angle Between Two Vectors	The angle between the two vectors is calculated by rearranging the scalar product formula				

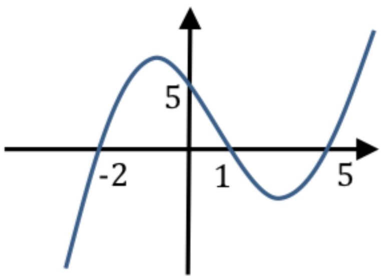
	$\cos\theta = \frac{\boldsymbol{a}.\boldsymbol{b}}{\lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert}$ which can be expanded to $$\cos\theta = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert}$$ Note: To find the angle between two vectors, the vectors must be pointing away from or towards each other. They must be going in the same direction Example:  				
Perpendicular Vectors	Vectors are perpendicular when **a**.**b** = 0				



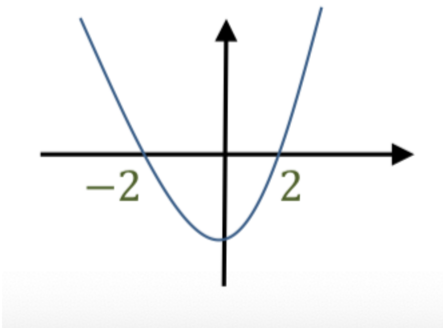
Relationships and Calculus

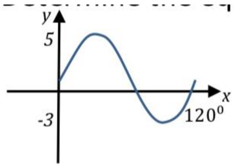
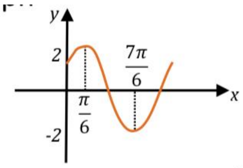
Topic	Skills	Notes			
Polynomials					

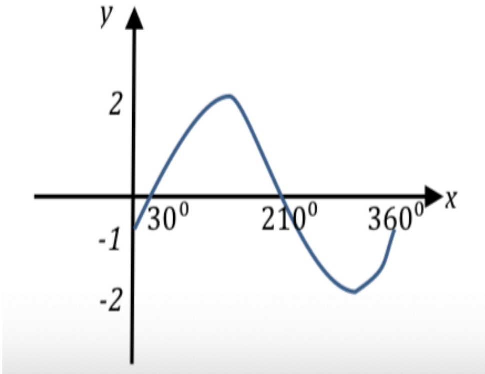
Fully Factorise a Polynomial	Use synthetic division Example: Factorise $f(x) = x^3 - 3x + 2$ Set up synthetic division using coefficients from polynomial. • If there is no term, use 0 • The value outside the division is derived from factors of the last term (in this case factors of 2) • If the remainder of the division is 0 then the value outside the division is a root  $\therefore (x+2) is\ a\ factor\ and\ x = -2\ is\ a\ root$ $(x+2)(x^2 - 2x + 1) = 0$ $(x+2)(x-1)(x-1) = 0$ $\therefore x = -1\ (twice)\ and\ x = 2$				
Remainder Theorem	Find the quotient and remainder of a function • Use synthetic division with the given value • The value at the end is the remainder Example: Find the quotient and remainder when $f(x) = (x^3 - 3x + 9)$ is divided by $(x - 3)$  $\therefore f(x) = (x^3 + 3x + 6)$ and remainder 27				

Identify the Equation of a Polynomial from a Graph	• Determine the factors from the roots on the graph • Set up a polynomial with a coefficient of k outside the brackets • Substitute the y-intercept or given point to determine the value of k Example:  $y = k(x+2)(x-1)(x-5)$ When x = 0, y = 5 $5 = k(0+2)(0-1)(0-5)$ $5 = 10k$ $k = \frac{1}{2}$ $\therefore y = \frac{1}{2}(x+2)(x-1)(x-5)$				
Show that a Term is a Factor of a Polynomial	• From the factor, determine the root • Use synthetic division with the given value • If the remainder is 0 the term is a factor				
Find Unknown Coefficients of a Polynomial	Substitute roots into equation and solve simultaneously Example: Find the values of p and q if $(x+2)$and $(x-1)$ are factors of $f(x) = x^3 + 4x^2 + px + q$ $(x+2) is\ a\ factor\ \therefore x = -2\ is\ a\ root$ $(-2)^3 + 4(-2)^2 - 2p + q = 0$ $8 - 2p + q = 0$ $q = 2p - 8$ $(x-1) is\ a\ factor\ \therefore x = 1\ is\ a\ root$				

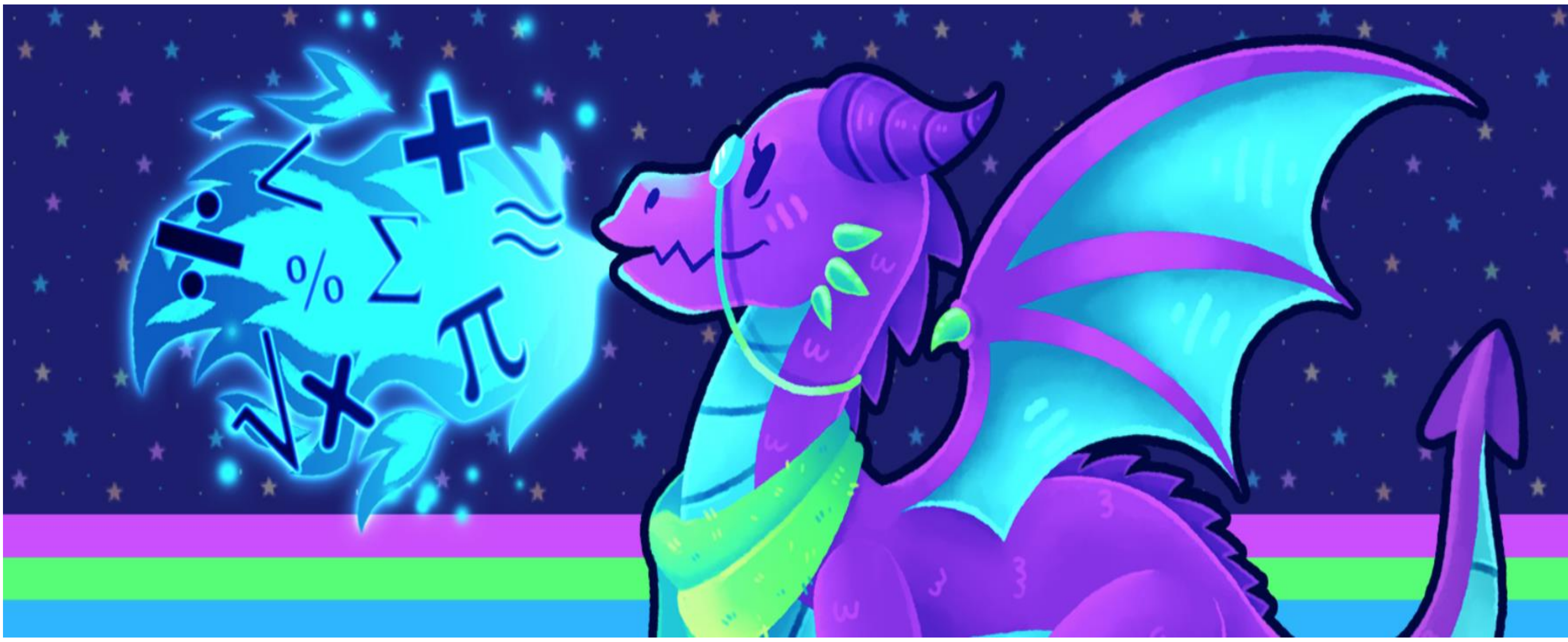
	$(1)^3 + 4(1)^2 + p + q = 0$ $5 + p + q = 0$ $q = -p - 5$ Solve two equations simultaneously $2p - 8 = -p - 5$ … … $\therefore p = 1, q = -6$ *and* $f(x) = x^3 + 4x^2 + x - 6$				
Sketch the Graph of a Polynomial Function	• Find the x-intercepts (roots, when y=0) using synthetic division • Find the y-intercept (when x=0) • Find stationary points and their nature • Find out negative and positive x				
Quadratic Functions					
Show a Line is a Tangent to a Quadratic Function	• Equate the line and quadratic • Bring to one side • Use the discriminant or factorise to show repeated root Example: Show that the line $y = x + 5$ is a tangent to the curve $y = x^2 + 5x + 9$ $x^2 + 5x + 9 = x + 5$ $x^2 + 4x + 4 = 0$ $(x + 2)(x + 2) = 0$ $x = -2$ *twice* $\therefore$ *line is a tangent to quadratic*				
Determine Nature of Intersection Between a Line and a Quadratic Function	• Equate line and quadratic • Bring to one side • Use the discriminant to determine nature of intersection				
Determine Points of Intersection Between a Line and a Quadratic Function	• Equate line and quadratic • Bring to one side • Solve for x • Substitute x values into line to find y values				

Use Discriminant to Find Unknown Coefficients of a Quadratic Function	• Identify coefficients a, b, and c • Use discriminant Example: Find p given that $x^2 + x + p = 0$ has real roots a = 1, b = 1, c = p $b^2 - 4ac \geq 0$ $1^2 - 4(1)(p) \geq 0$ $1 - 4p \geq 0$ $1 \geq 4p$ $p \leq \frac{1}{4}$ Example: Find p given that $4x^2 + 2px + 1 = 0$ has no real roots a = 4, b = 2p, c = 1 $b^2 - 4ac < 0$ $(2p)^2 - 4(4)(1) < 0$ $4p^2 - 16 < 0$ $4(p^2 - 4) < 0$ $4(p + 2)(p - 2) < 0$ Sketch graph  $for\ no\ real\ roots -2 < p < 2$				
Trigonometry					
Convert from Degrees to Radians	Multiply degrees by π and divide by 180 then simplify Example: Change 35° to radians $\frac{35\pi}{180} = \frac{7\pi}{36}$				
Convert from Radians to Degrees	Multiply radians by 180 and divide by π then simplify Example: Change $\frac{5\pi}{6}$ to degrees				

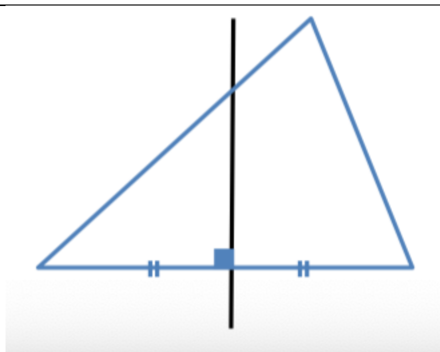
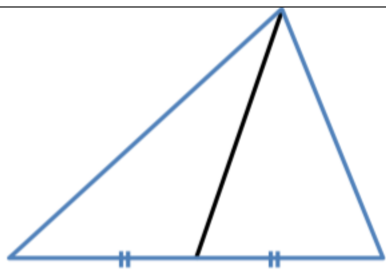
	$\frac{5\pi}{6} = \frac{5 \times 180\pi}{6\pi} = \frac{5 \times 30}{1} = 150°$				
Solve Trig Equations With Multiple Solutions	• Identify how many solutions from question • Solve the equation Example: Solve $2\cos 3x = 1$, for $0 \leq x \leq \pi$ $2\cos 3x = 1$ $\cos 3x = \frac{1}{2}$ $3x = \frac{\pi}{3}, \pi - \frac{\pi}{3}, 2\pi + \frac{\pi}{3}$ $3x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}$ $x = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{7\pi}{9}$				
Solving Trig Equations by Factorising	Factorise the equation in the same way as an algebraic equation, by looking for common factors, difference of two squares, or a trinomial Example: Solve $2\sin x \cos x + \sin x = 0$, for $0 \leq x \leq 180°$ Factorise: $\sin x\,(2\cos x + 1) = 0$ $\sin x = 0$ $2\cos x + 1 = 0$ $x = 0°, 180°$ $\cos x = -\frac{1}{2}$ Q2: $x =$ $120°$ $\therefore x = 0°, 120°, 180°$				
Identify the Equation of a Trig Function from its Graph	Example: Determine the equation of the graph **1.**  **2.**  **Ans:** $y = 4\sin 3x + 1$ **Ans:** $y = 2\cos(x - \frac{\pi}{6})$				

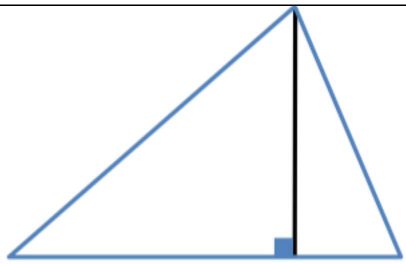
Sketch a Trig Graph from its Equation	Example: Sketch the graph of $y = 2\sin(x - 30)°$ for $0 \leq x \leq 360$ showing clearly where the graph cuts the x-axis and the y-axis Amplitude of 2. Graph moves 30° to the right. Find x-intercepts when y = 0. Find y-intercept when x = 0. 				
Further Calculus					
Differentiate Trig Equations	$y = \sin x$ $\quad\quad$ $y = \cos x$ $\frac{dy}{dx} = \cos x$ $\quad\quad$ $\frac{dy}{dx} = -\sin x$				
Integrate Trig Functions	$\int \sin x \; dx = -\cos x + C$ $\int \cos x \; dx = \sin x + C$				
Chain Rule	Used for differentiating composite functions • Differentiate the outer function • Multiply by the derivative of the inner function If $h(x) = f(g(x))$ then $h'(x) = f'\big(g(x)\big) \times g'(x)$ Example: Find $\frac{dy}{dx}$ when $y = \sqrt{2x - 5}$ Prepare function for differentiation $y = (2x - 5)^{\frac{1}{2}}$ $\frac{dy}{dx} = \frac{1}{2}(2x - 5)^{-\frac{1}{2}} \times 2$				

	$\frac{dy}{dx} = \frac{1}{\sqrt{2x-5}}$ Example: Find $\frac{dy}{dx}$ when $y = 3\,cos^2\,x$ Prepare function for differentiation $y = 3(\cos x)^2$ $\frac{dy}{dx} = 6(\cos x)^1 \times \sin x$ $\frac{dy}{dx} = 6\cos x \sin x$				
Integration of Composite Functions	When integrating composite functions • Integrate the outer function • Divide by the derivative of the inner function $$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{(n+1)\times a} + C$$ Example: $$\int (2x^3+5)^4\;dx$$ $$\int (2x^3+5)^4\;dx\;=\frac{(2x^3+5)^4}{5\times 6x^2} + C = \frac{(2x^3+5)^4}{30x^2} + C$$ Example: $$\int \sin(4x-3)dx$$ $$\int \sin(4x-3)dx = \frac{-\cos(4x-3)}{4} + C$$				



Applications

Topic	Skills	Notes			
Straight Line					
Equation of a Perpendicular Bisector	• Find the midpoint of the line joining the 2 points • Find gradient using perpendicular gradients • Substitute midpoint and inverted gradient into $y - b = m(x - a)$ 				
Equation of a Median	• Find the midpoint of the line joining the 2 points • Find gradient of the median 				

	• Substitute into $y - b = m(x - a)$				
Equation of an Altitude	• Find gradient of the altitude using  perpendicular gradients • Substitute into $y - b = m(x - a)$ with point from vertex				
Collinearity	• Show that 3 points are collinear • Find gradients and point in common • Statement: Points A, B, and C are collinear as $m_{AB} = m_{BC}$ and point B is common to both				
Circles					
Equation of Circle with Centre the Origin and Radius r	$x^2 + y^2 = r^2$				
Equation of a Circle with Centre (a,b) and Radius r	• Determine the centre and radius • Substitute into equation $(x - a)^2 + (y - b)^2 = r^2$				
Centre and Radius of a Circle from its Equation	Use the equation $x^2 + y^2 + 2gx + 2fy + c = 0$ Centre: $(-g, -f)$ Radius: $r = \sqrt{g^2 + f^2 - c}$ Note: If $g^2 + f^2 - c < 0$ the equation is not a circle				
Equation of a Tangent to a Circle	• Determine the gradient of the radius from the centre and the point of contact of the tangent • Find gradient using perpendicular gradients 				

	Substitute perpendicular gradient and point into equation of a line $y - b = m(x - a)$				
Points of Intersection of a Line and a Circle	• Rearrange the line to $y = mx + c$ then substitute into the equation of a circle • Solve the quadratic to find x • Substitute x value into $y = mx + c$ to find y				
Use Discriminant to Determine Whether a Line and Circle Intersect	• Rearrange the line to $y = mx + c$ then substitute into the equation of a circle • Simplify to quadratic form • Use discriminant to determine intersection				
Recurrence Relations					
Form Linear Recurrence Relations	Find values of a and b for relation $u_{n+1} = au_n + b$ Where a is the percentage multiplier and b is the increase				
Use Linear Recurrence Relations to Find Values	Start with u_0 (initial value) and substitute into relation				
Find the Limit of a Linear Recurrence Relation	• Determine values of a and b • Ensure $-1 < a < 1$ • Use limit formula $L = \frac{b}{1-a}$ • Interpret what the limit means in a specific context				
Differentiation					
Differentiate a Function	To differentiate $f(x) = ax^n$, $f'(x) = anx^{n-1}$ For $f(x) = g(x) + h(x), f'(x) = g'(x) + h'(x)$				
Find the Gradient or Rate of Change of a Function at a Given Point	• Know that $f'(x) = m = rate\ of\ change$ • Differentiate the function • Substitute x-coordinate into derivative Example: Find the gradient of $f(x) = 2x^3$ when $x = 1$ $f'(x) = 6x^2$ $f'(1) = 6(1)^2 = 6$				

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<tr><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Find the Equation of a Tangent to a Curve</td><td>

- Differentiate function
- Find gradient from derivative
- Substitute point and gradient into equation of a line

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<tr><td>Find Stationary Points and Determine their Nature</td><td>

Find Stationary Points

- Differentiate function
- Set equal to 0
- Solve to find x-coordinates of stationary points
- Substitute x-coordinates into original equation to find y-coordinates

Determine Nature

- Draw nature table with x values slightly above and below the stationary points
- Sketch the nature from positive or negative values

<table>
<tr><td>x</td><td>3^-</td><td>3</td><td>3^+</td></tr>
<tr><td>$f'(x)$</td><td>-</td><td>0</td><td>+</td></tr>
<tr><td>slope</td><td>\</td><td>_</td><td>/</td></tr>
</table>

</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Sketch a Curve</td><td>

- Find stationary points and nature
- Find roots by solving function (when y=0)
- Find y-intercept (when x=0)
- Find large positive and large negative x
- Sketch information on graph

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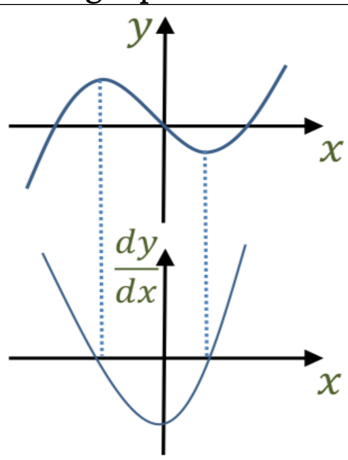
<tr><td>Sketch the Derived Function</td><td>

- Sketch function
- Extend stationary points to other coordinate axis
- Determine where the gradient is +ve and -ve



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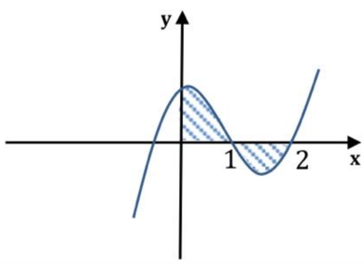
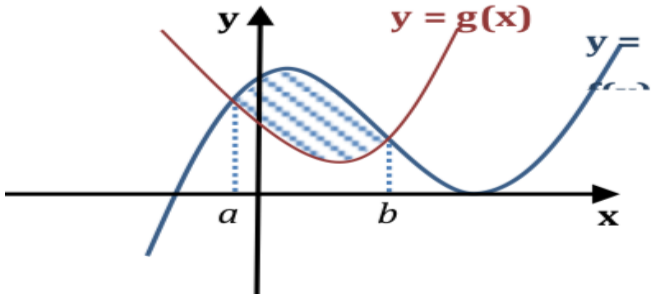
Closed Intervals	Find the maximum and minimum value in a closed interval • Find the stationary points and determine their nature • Find the y-coordinates at the extents of the interval • Examine to see where the maximum and minimum values are				
Increasing and Decreasing Functions	• Differentiate the function • Determine where gradient is positive or negative from the derivative or a sketch of the derivative Note: A function is increasing where the gradient m is positive and decreasing where the gradient is negative				
Integration					
Integrate a Function	$$\int ax^n \, dx = \frac{ax^{n+1}}{n+1} + C$$				
Evaluate a Definite Integral	Example: Evaluate $\int_1^3 4x \, dx$ $$\int_1^3 4x \, dx = (2(3)^2) - (2(1)^2) = 16$$				
Area Between Curve and x-axis	• Find the area above the x-axis • Find the area below the x-axis (ignore the negative) • Add them together Example:  $$\int_0^1 f(x)dx$$ $$\int_1^2 f(x)dx$$				

Area Between 2 Curves	 • Set the curves equal to each other and solve to find the limits • Set up the integral with $$\int_a^b [f(x) - g(x)]\,dx$$				
Differential Equations	Equations of the form $\frac{dy}{dx} = ax + b$ are called differential equations. They are solved by integration. Example: The curve $y = f(x)$ is such that $\frac{dy}{dx} = 9x^2$, the curve passes through (1, 5). Express y in terms of x. $$y = \int 9x^2\,dx = 3x^3 + C$$ At (5, 1) $5 = 3(1)^3 + C$ $5 = 3 + C$ $C = 2$ $$\therefore y = 3x^3 + 2$$				