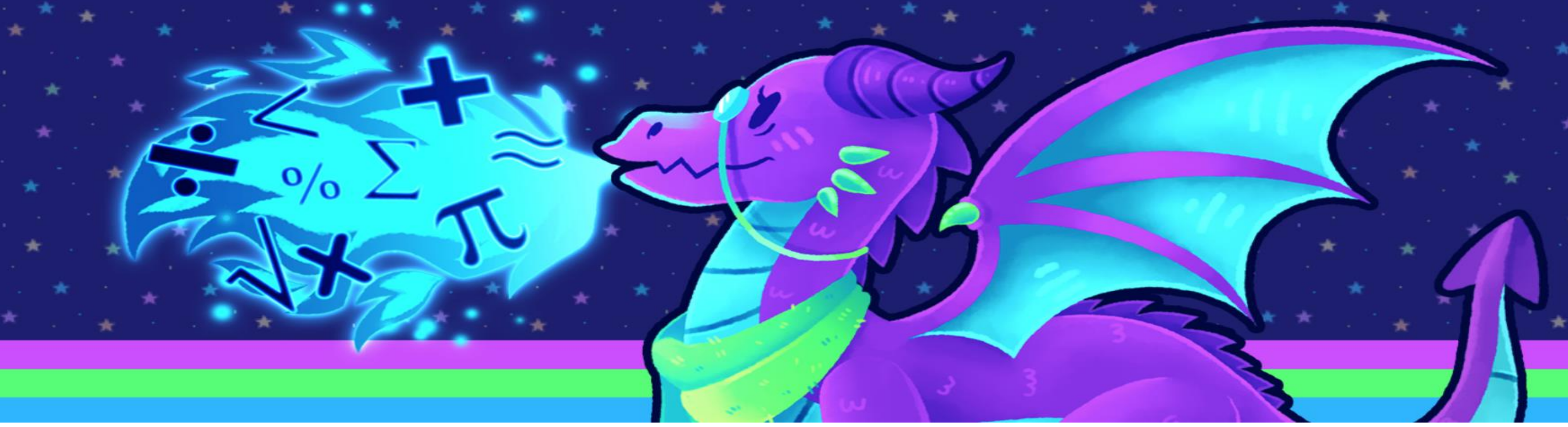


Baldragon Academy

National 5 Maths

Checklist

Contents: **Page**

Expressions and Formulae

Topic	Skills	Notes			
Rounding					
Round to Decimal places	Example: 25.1241 = 25.1 to 1 d.p. Example: 34.676 = 34.68 to 2 d.p.				
Round to Significant Figures	Example: 1276 = 1300 to 2 sig figs Example: 0.06356 = 0.064 to 2 sig figs Example: 37,684 = 37,700 to 3 sig figs Example: 0.005832 = 0.00583 to 3 sig figs				
Surds					
Simplifying	Learn Square Numbers: 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169. Use square numbers as factors: Example: $\sqrt{72} = \sqrt{36} \times \sqrt{2} = 6\sqrt{2}$				
Add/ Subtract	Example: $\sqrt{72} + \sqrt{50} = \sqrt{36} \times \sqrt{2} + \sqrt{25} \times \sqrt{2}$ $= 6\sqrt{2} + 5\sqrt{2}$ $= 11\sqrt{2}$				
Multiply/ Divide	Example:				

	$\sqrt{5} \times \sqrt{15}$ $= \sqrt{5 \times 15}$ $= \sqrt{75}$ $= \sqrt{25} \times \sqrt{3}$ $= 5\sqrt{3}$ Example: $\frac{\sqrt{48}}{\sqrt{3}}$ $= \sqrt{\frac{48}{3}}$ $= \sqrt{16}$ $= 4$				
Rationalise Denominator	Remove surd from denominator Example: $\frac{1}{\sqrt{3}} = \frac{1 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{\sqrt{3}}{3}$				
Indices					
Use Laws of Indices	1. $a^x \times a^y = a^{x+y}$ 2. $a^x \div a^y = a^{x-y}$ 3. $(a^x)^y = a^{xy}$ 4. $\frac{1}{a^x} = a^{-x}$ 5. $a^0 = 1$				
Scientific Notation/ Standard Form	The first number is always between 1 and 10. Example: $54{,}600 = 5.46 \times 10^4$ Example: $0.000978 = 9.78 \times 10^{-4}$				
Evaluate using Indices	Example: $27^{\frac{2}{3}} = \sqrt[3]{27^2} = 3^2 = 9$				
Algebra					

Expand Single Brackets	Example: $3(x + 4)$ $= 3x + 12$				
Expand Two Brackets	Use FOIL (Front, Outside, Inside, Last) Example: $(x + 3)(x - 2)$ $= x^2 + 3x - 2x - 6$ $= x^2 + x - 6$				
	Know that every term in the first bracket must multiply every term in the second. Example: $(x + 2)(x^2 - 3x - 4)$ $= x^3 - 3x^2 - 4x + 2x^2 - 6x - 8$ $= x^3 - x^2 - 10x - 8$				
Simplify Expression	Put together the terms that are the same: Example: $x^2 + 4x + 3 - 2x + 8$ $= x^2 + 2x + 11$ Example: $a \times a \times a$ $= a^3$				
Factorise – Highest Common Factor (HCF)	Take the factors each term has in common outside the bracket. Example: $4x^2 + 8x$ $= 4x(x + 2)$ Note: always look for a common factor first				
Factorise – Difference of Two Squares	One square number take away another Example: $x^2 - 9$ $= (x + 3)(x - 3)$				

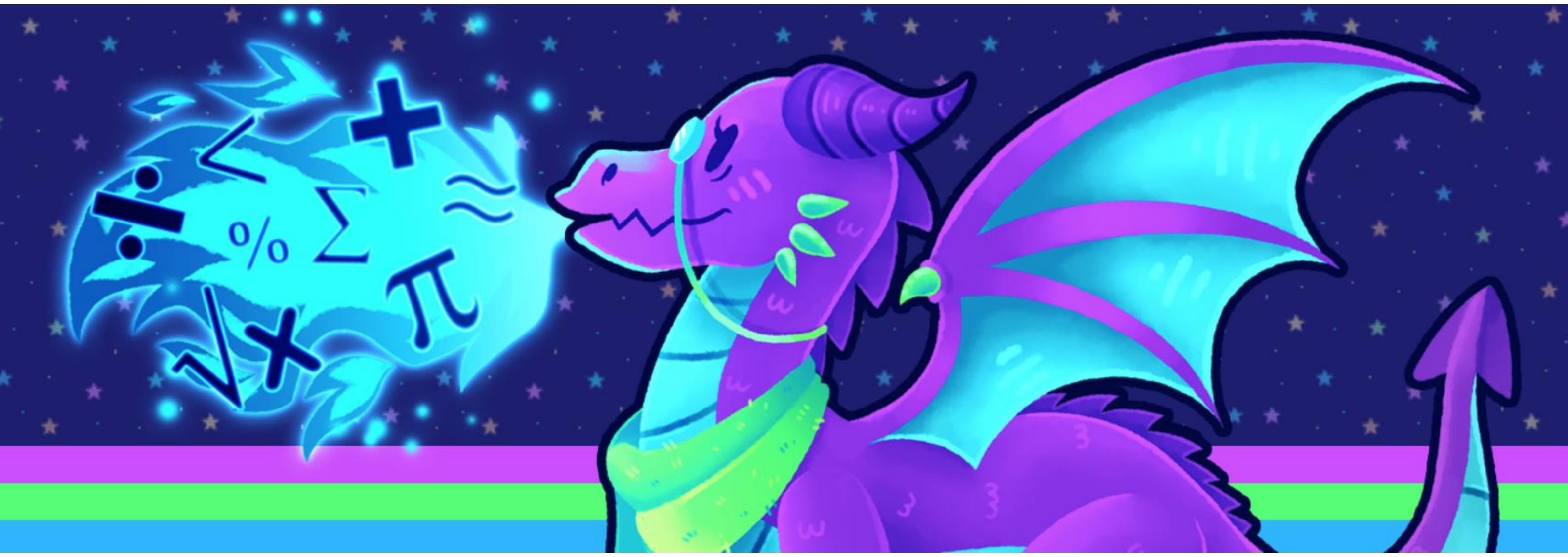
	Example: $5x^2 - 125$ $= 5(x^2 - 25)$ (pull out HCF first) $= 5(x + 5)(x - 5)$				
Factorise – Trinomial (no coefficient)	2 numbers that multiply to give the value of c and add to give the value of b $ax^2 + bx + c$ Example: $x^2 - x - 6$ $= (x - 3)(x + 2)$				
Factorise – Trinomial (coefficient)	$ax^2 + bx + c$ multiply a and c and find 2 numbers that multiply to give this answer but add to give b Example: $3x^2 - 13x - 10$ $= 3x^2 - 15x + 2x - 10$ $= 3x(x - 5) + 2(x - 5)$ $= (x - 5)(3x + 2)$				
Complete the Square	Example: $x^2 + 8x - 13 = (x + 4)^2 - 13 - 16$ $= (x + 4)^2 - 29$				

Algebraic Fractions

Simplifying Algebraic Fractions	**Step 1:** Factorise expressions **Step 2:** Look for common factors. **Step 3:** Cancel and simplify Example: $\frac{6x^2 - 12x}{x^2 + x - 6} = \frac{6x(x-2)}{(x+3)(x-2)} = \frac{6x}{x+3}$				
Add and Subtract Fractions	Need common denominator Kiss Kiss Smile Example:				

	$\frac{5x}{b}+\frac{3d}{2c}=\frac{10xc}{2bc}+\frac{3bd}{2bc}=\frac{10xc+3bd}{2bc}$				
Multiply Fractions	Multiply the numerators and the denominators together Example: $\frac{2x}{3y}\times\frac{3d}{5m}=\frac{6xd}{15ym}=\frac{2xd}{5ym}$				
Divide Fractions	KFC Keep the first fraction Flip the second fraction Change the divide to a multiply Example: $\frac{6x^2}{7y}\div\frac{4x}{3z}=\frac{6x^2}{7y}\times\frac{3z}{4x}=\frac{18x^2z}{28xy}=\frac{9xz}{14y}$				
Volumes					
Volume of a cylinder	$V=\pi r^2h$				
Rearrange each of the formulae to find an unknown	Example: Cylinder has volume 400 cm^3 and radius 6 cm, calculate the height $V=\pi r^2h$ $400=\pi\times 6^2$ $h=\frac{400}{\pi\times 6^2}$ height = 3.5 cm				
Gradient					
Find the gradient of a line joining 2 points	The gradient is represented by the letter m **Step 1:** Select 2 coordinates **Step 2:** Label them (x_1, y_1) and (x_2, y_2) **Step 3:** $m=\frac{y_2-y_1}{x_2-x_1}$ Example: (-4, 4) and (12, - 28)				

	x_1 y_1 x_2 y_2 $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-28) - 4}{12 - (-4)} = \frac{-32}{16} = -2$				
Circles					
Length of Arc	This find the length of the arc of a sector of a circle $arc\ length = \frac{angle}{360°} \times \pi D$				
Arc of Sector	$sector\ area = \frac{angle}{360°} \times \pi r^2$				



Relationships

Topic	Skills	Notes			
Straight Line					
Gradient	• Represented by m • Measure of steepness of slope • + gradient = line increasing				

	• - gradient = line decreasing				
y - intercept	• Represented by c • Shows where the line cuts the y – axis • Find by setting x = 0				
Gradient	The gradient is represented by the letter m **Step 1:** Select 2 coordinates **Step 2:** Label them (x_1, y_1) and (x_2, y_2) **Step 3:** $m = \frac{y_2 - y_1}{x_2 - x_1}$ Example: (-4, 4) and (12, - 28) x_1 y_1 x_2 y_2 $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-28) - 4}{12 - (-4)} = \frac{-32}{16} = -2$				
Find equation of a line	**Step 1:** Find gradient m **Step 2:** Find y-intercept c **Step 3:** Substitute into y = mx + c				

Solving Equations/ Inequations

Solving Equations	Use suitable method: Example: 5(x + 4) = 2(x – 5) 5x + 20 = 2x – 10 5x = 2x – 30 3x = - 30 x= - 10				
Solving Inequations	Solve the same way as equations. Note: When dividing by a negative change the sign Example: $-3x < 15$ $x > -5$				

Simultaneous Equations

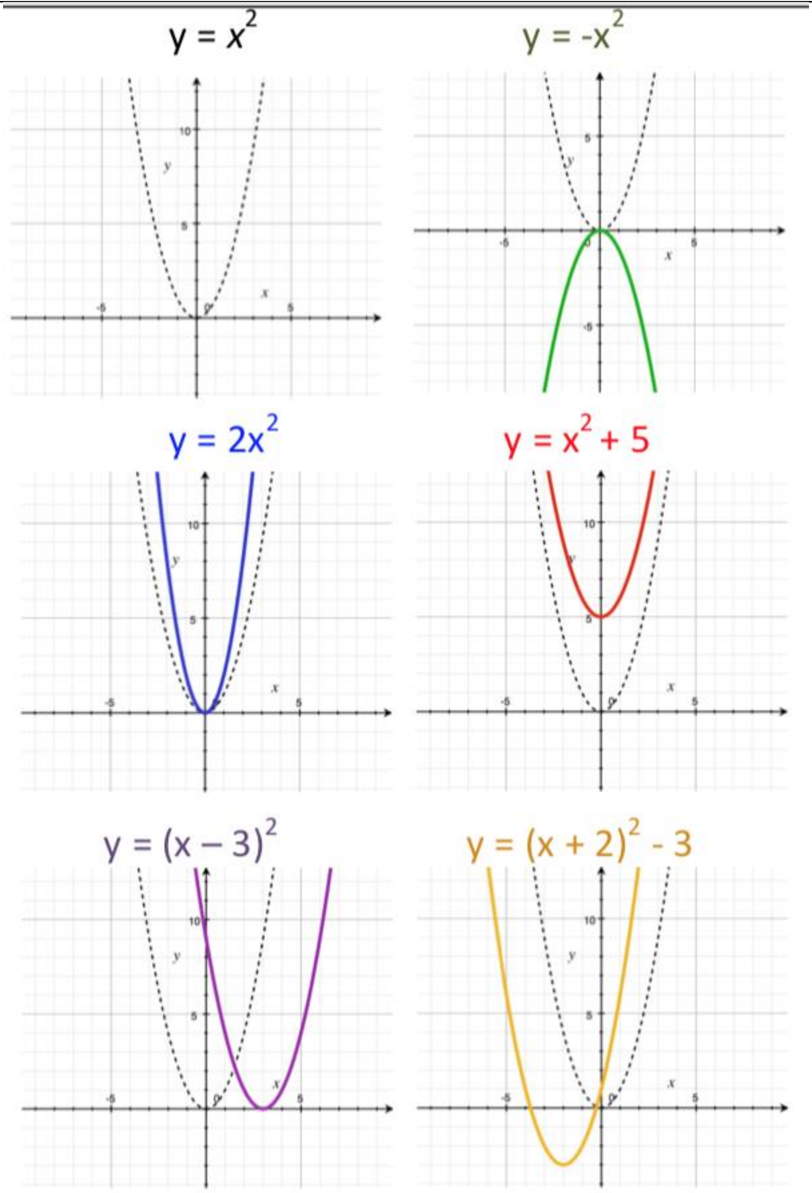
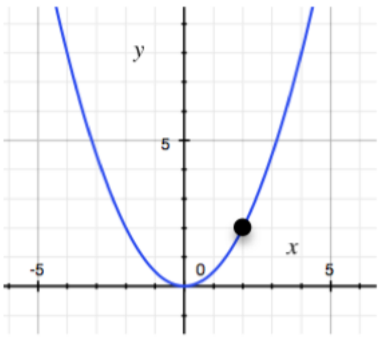
Solve by sketching lines	**Step 1:** Rearrange formula if needed to y = mx + c **Step 2:** Sketch lines using table of points **Step 3:** Find coordinate of point of intersection				
Solve by elimination	**Step 1:** Scale equations to make one unknown equal **Step 2:** Add or subtract equations to eliminate term and solve. **Step 3:** Substitute number to find second term. Example: 4a + 3b = 7 → **1** 2a − 2b = -14 → **2** **1** × 2 , **2** × 3 8a + 6b = 14 → **3** 6a − 6b = - 42 → **4** **3** + **4** 14a = - 28 a = -2 substitute a = -2 into **1** 4(-2) + 3b = 7 -8 + 3b = 7 3b = 15 b = 5				

Change the Subject

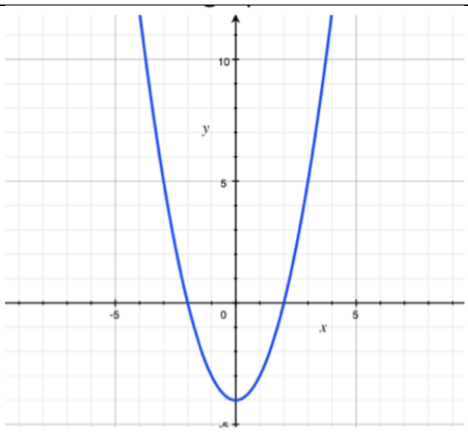
Linear Equations	Rearrange equations to change the subject: Example: $D = 4C - 3$ [**C**] $4C - 3 = D$ $4C = D + 3$ $C = \frac{D+3}{4}$				

Equations with powers or roots	Example: $V = \pi r^2 h$ **[r]** $\pi r^2 h = V$ $r^2 = \dfrac{V}{\pi h}$ $r = \sqrt{\dfrac{V}{\pi h}}$				

Quadratic Functions

Quadratics and their equations					
Equations of quadratics $y = kx^2$	**Step 1:** Identify coordinate from graph **Step 2:** Substitute into $y = kx^2$ **Step 3:** Solve to find k Example: Coordinate: (2, 2) Substitution: $2 = k(2)^2$ $2 = 4k$ $k = 0.5$ 				
Sketching Quadratics $y = k(x + a)^2 + b$	**Step 1:** Identify shape, if k is + then graph is a happy face (min turning point), if k is a – then graph is sad face (max turning point) **Step 2:** Identify turning point (-a, b) **Step 3:** Sketch axis of symmetry x = -a **Step 4:** Find y – intercept (make x=0) **Step 5:** Sketch information				

Sketching Quadratics $y = (x + a)(x - b)$	**Step 1:** Identify shape (+ or -) **Step 2:** Identify roots (x – intercepts) $x = -a$, $x = b$ **Step 3:** Find y-intercept (make $x=0$) **Step 4:** Identify turning point Example: $y = (x + 4)(x - 2)$ + graph so min turning point roots: $x = -4$, and $x = 2$ y-intercept: $y = (0 + 4)(x - 2) = -8$ turning point: halfway between roots so $x = -1$ sub $x = -1$ into $y = (x + 4)(x - 2)$ $= (-1 + 4)(-1 - 2)$ $= (3)(-3)$ $= -9$ so turning point = $(-1, -9)$				
Solving Quadratics (finding roots)- algebraically	**Step 1:** Equate to zero **Step 2:** Factorise quadratic **Step 3:** Set each factor equal to zero **Step 4:** Solve each factor to find roots Example: $y = x^2 - 5x - 6$ $(x - 6)(x + 1) = 0$ $x - 6 = 0$ and $x + 1 = 0$ $x = 6$ and $x = -1$				
Solving Quadratics (finding roots) - graphically	Read roots from graph				

	 $x = 2, \ x = -2$				
Solving Quadratics – Quadratic Formula	When asked to solve a quadratic to a number of decimal places use the quadratic formula: $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ where $y = ax^2 + bx + c$ Example: solve $y = x^2 - 6x + 2$ to 1 d.p. $a = 1$, $b = -6$, $c = 2$ $$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 1 \times 2}}{2 \times 1}$$ $$x = \frac{6 \pm \sqrt{28}}{2}$$ $$x = \frac{6 + \sqrt{28}}{2} \qquad x = \frac{6 - \sqrt{28}}{2}$$ $$x = 5.6 \qquad x = 0.4$$				
Discriminant	**$b^2 - 4ac$** where $y = ax^2 + bx + c$ The discriminant describes the nature of the roots $b^2 - 4ac > 0$ means 2 real roots $b^2 - 4ac = 0$ means equal roots $b^2 - 4ac < 0$ means no real roots Example: Determine the nature of the roots of the quadratic $y = x^2 + 5x + 4$				

$a = 1$, $b = 5$, $c = 4$

$$b^2 - 4ac = 5^2 - 4 \times 1 \times 4$$
$$= 25 - 16$$
$$= 9$$

Since $b^2 - 4ac > 0$ means 2 real roots

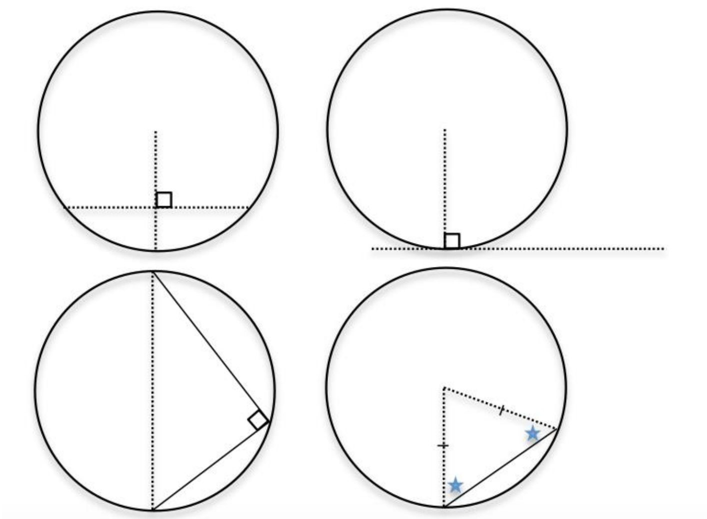
Example:

Determine p, where $x^2 + 8x + p$ has equal roots

$$b^2 - 4ac = 0$$
$$8^2 - 4 \times 1 \times p = 0$$
$$64 - 4p = 0$$
$$-4p = -64$$
$$p = 16$$

Properties of Shapes

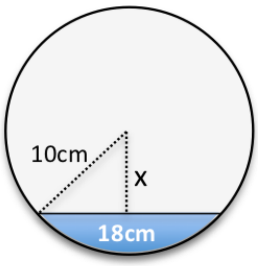
Circles



Pythagoras

Use Pythagoras Theorem to solve problems involving circles and 3D shapes.

Example:



Find the depth of water in a pipe of radius 10 cm.

r is the radius

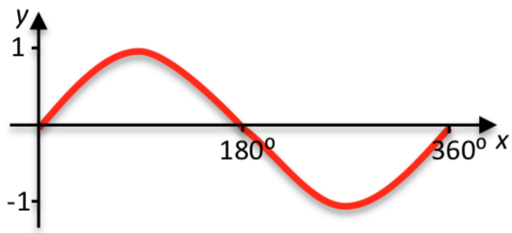
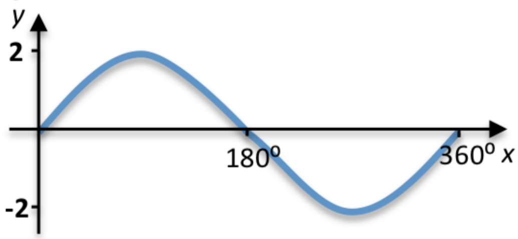
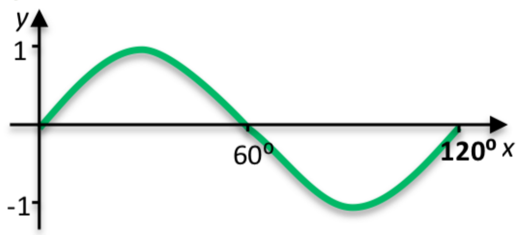
$$x^2 = 10^2 - 9^2$$
$$x^2 = 19$$
$$x = 4.4$$

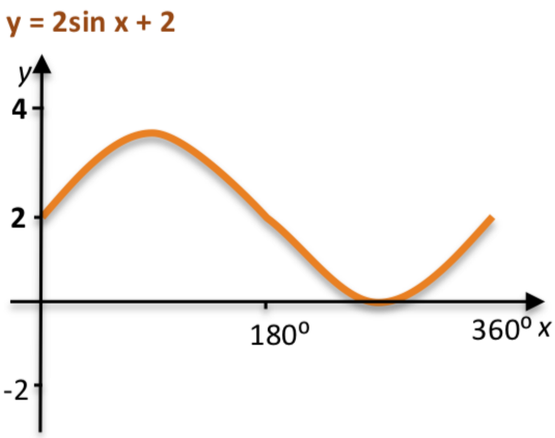
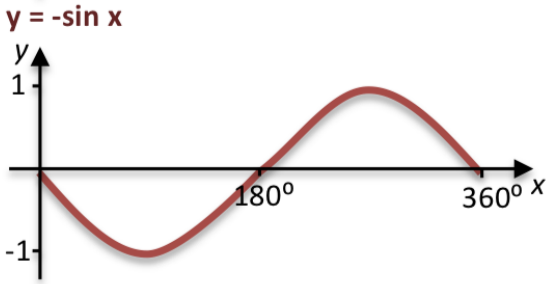
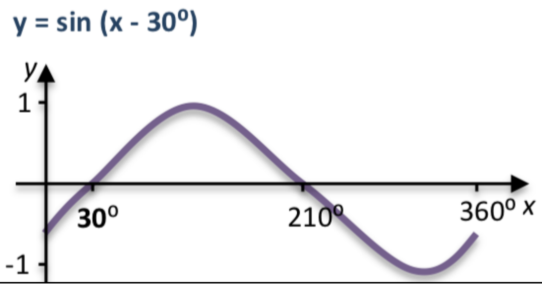
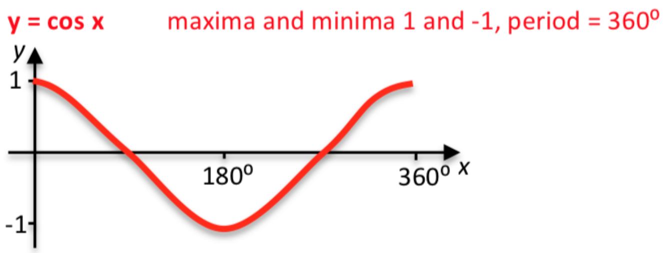
Depth = 10 – 4.4 = 5.6 cm

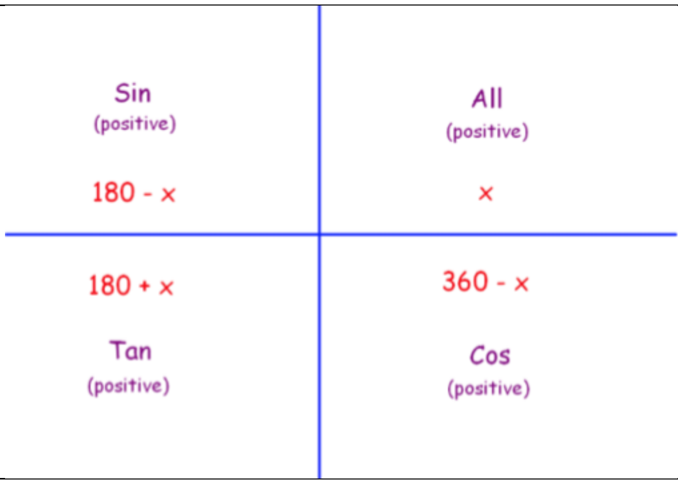
Similar Shapes

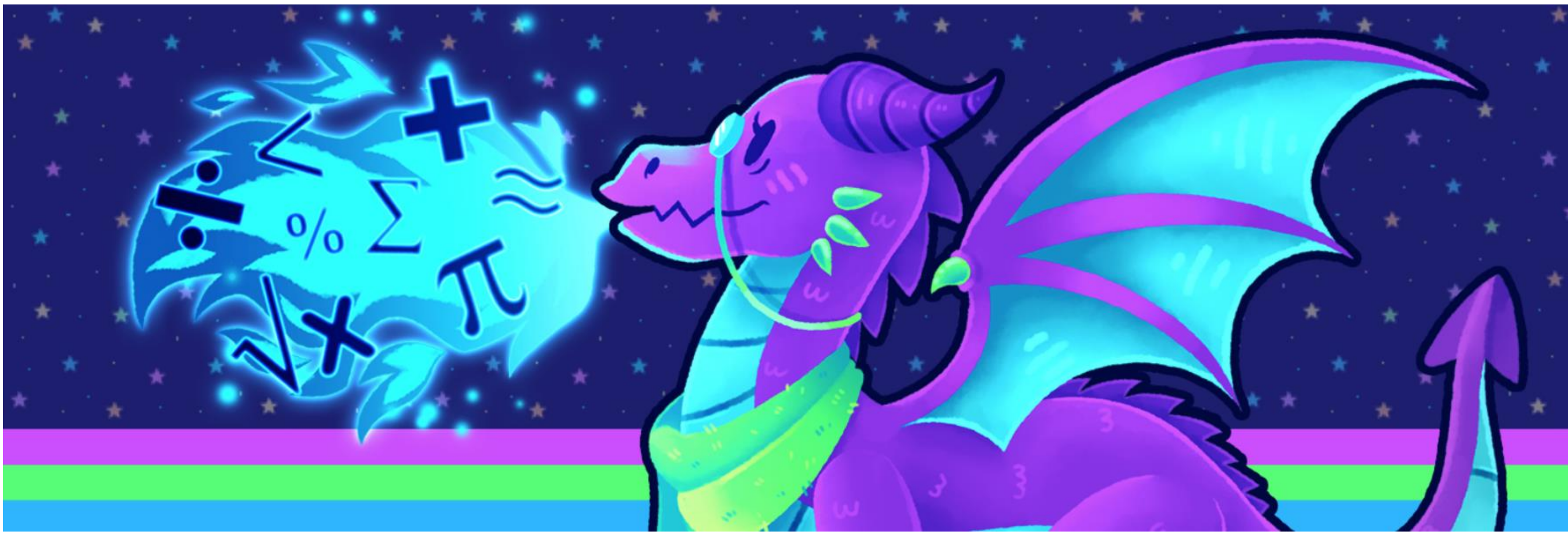
Linear Scale Factor	$Linear\ Scale\ Factor = \frac{New\ Length}{Original\ Length}$				
Area Scale Factor	$Area\ Scale\ Factor = (\frac{New\ Length}{Original\ Length})^2$				
Volume Scale Factor	$Volume\ Scale\ Factor = (\frac{New\ Length}{Original\ Length})^3$				

Trigonometry

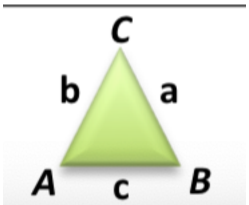
Trig Graphs – Sine Curve	y = asinb(x + d) + c a = how much the sin graph has stretched along the y – axis b = no. of waves between 0 and 360 c = movement of graph vertically d = movement of graph horizontally Example: **y = sin *x*** maxima and minima 1 and -1, period = 360^o  **y = 2sin x**  **y = sin 3x** 				

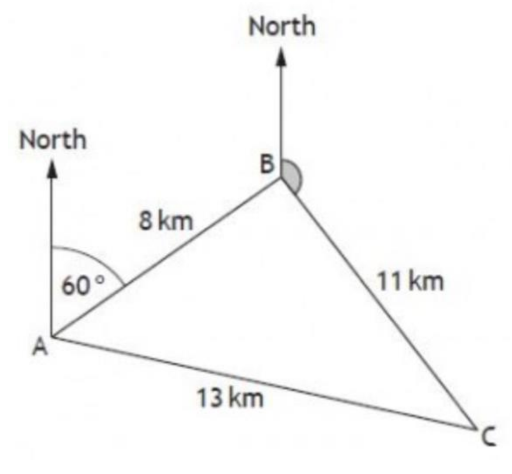
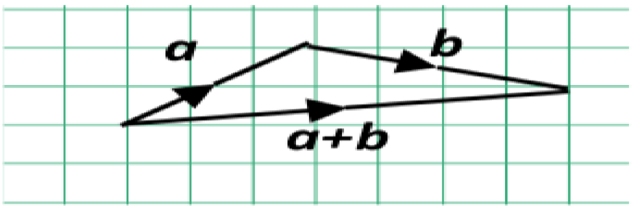
	  				
Trig Graphs – Cosine Curve	y = asinb(x + d) + c Example:  The same transformations apply to the cosine graph as they do for the sine graph				

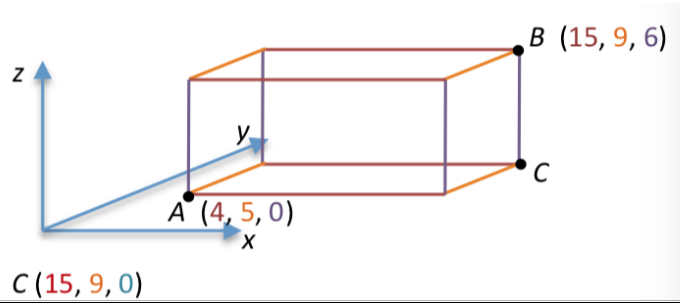
CAST Diagram	 				
Solving Trig Equations	Use the diagram above to solve trig equations: Example: Solve 2sinx – 1 = 0 2sinx = 1 sinx = 0.5 Since sin is positive we are in Q1 and Q2 Q1: $x = \sin^{-1}(0.5)$ $= 30°$ Q2: 180° – 30° = 50° Example: Solve 4tanx + 5 = 0 4tanx = - 5 tanx = -1.25 Since tan is negative we are in Q2 and Q4 Q1: $x = \tan^{-1}(1.25)$ $= 51.3°$ Q2: x = 180° – 51.3° = 128.7° Q4: x = 360° – 51.3° = 308.7°				
Trig Identities	Know: $sin^2\theta + cos^2\theta = 1$ $\tan\theta = \frac{\sin\theta}{\cos\theta}$				



Applications

Topic	Skills	Notes			
Triangle Trigonometry					
Triangle	Label Triangle 				
Area of a triangle	$A = \frac{1}{2} absinC$				
Sine Rule	$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$				
Cosine Rule	**side:** $a^2 = b^2 + c^2 - 2bccosA$				
	angle: $cosA = \frac{b^2 + c^2 - a^2}{2bc}$				
Bearings	Use knowledge of bearings to solve trig problems including knowledge of corresponding, alternate, and supplementary angles				

					
Vectors					
2D Line Segments	Add or subtract 2D line segments - Vectors end to end - Arrows in same direction 				
Position Vectors	The position vector of a coordinate is the vector from the origin to the coordinate Example: A (4, -3) has the position vector **a** = $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$				
Finding a Vector from 2 coordinates	To find a vector between 2 points, A and B $\vec{AB}$ = **b** – **a** Example: Calculate $\vec{AB}$ where A = (3, 2) and B = (7, 6) $\vec{AB}$ = **b** – **a** $= \begin{pmatrix} 7 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$				

3D Vectors	Determines coordinates of a point from a diagram representing a 3D object Look at difference in x, y, and z axes individually Example:  				
Vector Components	Add and Subtract 2D and 3D vector components Example: $\mathbf{a} = \begin{pmatrix}1\\1\\4\end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix}3\\2\\5\end{pmatrix}$, $\mathbf{a} + \mathbf{b} = \begin{pmatrix}1+3\\1+2\\4+5\end{pmatrix}$ $= \begin{pmatrix}4\\3\\9\end{pmatrix}$				
	Multiply vector components by a scalar Example: $2\mathbf{a} = 2\begin{pmatrix}1\\1\\4\end{pmatrix} = \begin{pmatrix}2\\2\\8\end{pmatrix}$				
	Find the magnitude of a 2D or 3D vector: Example: For vector $\mathbf{u} = \begin{pmatrix}1\\2\\3\end{pmatrix}$ $\lvert\mathbf{u}\rvert = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$				

Percentages

Compound Interest	Calculate multiplier from percentage then use multiplier to calculate appreciation/ depreciation Example: £500 with 5% interest for 3 years 100% + 5% = 105% = 1.05 $500 \times (1.05^3) = £578.81$				
Percentage Increase / Decrease	$\% \ increase \ or \ decrease = \frac{difference}{original} \times 100$				
Reverse Percentages	Find initial amount after a percentage increase or decrease. Calculate 1% or other easy percentage to find and multiply up to 100%. Example: A watch has been reduced by 30% to £42. What was it's original price? 70% = £42 (divide by 7 to find 10%) 10% = £6 (multiply by 10 to find 100%) 100% = £60 Original price was £60				

Fractions

Add and Subtract Fractions	Find a common denominator Kiss Kiss Smile Example: $\frac{2}{3} + \frac{4}{5} = \frac{10}{15} + \frac{12}{15} = \frac{22}{15}$				
Add and Subtract Mixed Fractions	Make improper fractions. Then add or subtract as normal. Example:				

	$2\frac{2}{3} + 3\frac{4}{5} = \frac{8}{3} + \frac{19}{5} = \frac{40}{15} + \frac{57}{15} = \frac{97}{15}$				
Multiply Fractions	Multiply top with top, and bottom with bottom Example: $\frac{3}{7} \times \frac{4}{5} = \frac{12}{35}$				
Multiply Mixed Fractions	Make top heavy fraction then as above: Example: $3\frac{3}{7} \times \frac{4}{5} = \frac{23}{7} \times \frac{4}{5} = \frac{92}{35}$				
Divide Fractions	KFC Keep the first fraction Flip the second fraction Change the divide to a multiply Example: $\frac{6}{7} \div \frac{2}{3} = \frac{6}{7} \times \frac{3}{2} = \frac{18}{10} = \frac{9}{5}$				
Statistics					
Mean	$\bar{x} = \frac{\sum n}{n} = \frac{sum\ of\ data}{number\ of\ terms}$				
Five-Figure Summary	l = lowest term Q_1 = lower quartile Q_2 = median Q_3 = upper quartile h = highest term				
Semi-Interquartile Range	$SIQR = \frac{Q_3 - Q_1}{2}$				
Standard Deviation	$SD = \sqrt{\frac{\sum(x-\bar{x})}{n-1}}$				
Comparing Data	Always compare the measure of average and the measure of spread.				

	Example: On average, Stacey runs more because her mean running time is greater, but Steve is more consistent as his standard deviation is smaller.				
Line of Best Fit	Use knowledge of straight line to find equation. Use equation to estimate unknown value				