



# Baldragon Academy Numeracy Techniques

# Calculating methods

## Addition

#### Example 534 + 2678

Place the digits in the correct "place value" columns with the numbers under each other. Begin adding in the ones column.

Show any carrying in the next column.

	Th	Н	Т	0
		5	3	4
+	<mark>1</mark> 2	<u>1</u> 6	<u>1</u> 7	8
	3	2	1	2

## Subtraction

#### Example: 7689 - 749

Place the digits in the correct "place value" columns with the numbers under each other.

Begin subtracting in the ones column.

You can't subtract 9 from 6 so move 1 ten from the 8 tens to the 6 ones to make 16 ones.

Note that the same has happened with the hundreds.

## **Multiplication**

#### Example 56 x 34

Separate the 56 and 34 into tens and ones.

Multiply the columns with the rows and place the answers in the grey boxes.

Add the numbers: 1500 + 180 + 200 + 24 = **1904** 

×	50	6
30	1500	180
4	200	24

	6	9	3	7
-		7	4	9
	67	<sup>1</sup> 6	78	<b>1</b> 6
	Th	Н	Т	0

### Division

**Example:** 432 ÷ 15

#### Long division method

		2	8	4 is not divisible by 15, so you divide 43 by 15.
15	4	3	2	3 x 15 = 45 which is more than 43 so choose <mark>2</mark> x 15 = <mark>30</mark> .
	3	0	$\downarrow$	Subtract 30 from 43 to give a remainder of 13.
	1	3	2	Bring the <b>2</b> down in order to make <b>132</b> .
	1	2	0	<b>8</b> × 15 = <b>120</b> .
		1	2	Subtract 120 from 132 to give a remainder of 12.
	28	r	12	There are no more numbers to bring down, therefore, the answer is: 28 with a remainder of 12.
	20		-	Lo with a remainder of 1L.

#### Concise method

					4 is not divisible by 15, so you divide 43 by 15.
		2	8	r12	3 x 15 = 45 which is more than 43 so choose
15	4	3	<sup>13</sup> 2	-	<mark>2</mark> × 15 = <mark>30</mark> .
					Subtract 30 from 43 to give a remainder of 13.
					Write 13 in front of the 2 to give 132.
					<b>8</b> × 15 = 120.
					Subtract 120 from 132 to give a remainder of 12.
					Therefore, the answer is: 28 r 12

#### **Division without tears!**

×	15	Sum		
10	150	150		
10	150	300		
5	75	375		
2	30	405		
1	15	420		
28		r12		

We use the 10, 5, 2 and 1 tables which are easier. 10 x 15 = 150 which is a great deal less than 432. Another 10 x 15 will make a total of 300. Another 10 x 15 will give a total of 450 which is more than 432 so we use  $5 \times 15 = 75$  giving a total of 375.  $5 \times 15$  is too big, so use  $2 \times 15 = 30$  gives 405.  $2 \times 15$  is too big, so use  $1 \times 15 = 15$  to give 420.  $1 \times 15$  is too big, therefore, the remainder is 432 - 420 = 12. By adding the "x" column we can see how many  $15_3$  there are in 432. Answer: 10 + 10 + 5 + 2 + 1 = 28.

#### Even numbers

2, 4, 6, 8, 10, 12, ..... 2 divides exactly into every even number.

#### Odd numbers

1, 3, 5, 7, 11, ..... 2 doesn't divide exactly into odd numbers.

#### Triangular numbers

1	= 1
1 + 2	= 3
1 + 2 + 3	<b>= 6</b>
1 + 2 + 3 + 4	= 10
1 + 2 + 3 + 4 + 5	= 15
1 + 2 + 3 + 4 + 5 + 6	= 21
1 + 2 + 3 + 4 + 5 + 6 + 7	= 28

The first seven triangular numbers are: 1, 3, 6, 10, 15, 21, 28

#### Prime numbers

A prime number has exactly **two** factors namely 1 and itself.

The factors of 17 are 1 and 17, therefore 17 is a prime number.

The prime numbers between 1 and 100 are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97 Note: 1 is not a prime number!

#### Square numbers

 $1^{2} = 1 \times 1 = 1$   $2^{2} = 2 \times 2 = 4$   $3^{2} = 3 \times 3 = 9$   $4^{2} = 4 \times 4 = 16$   $5^{2} = 5 \times 5 = 25$   $6^{2} = 6 \times 6 = 36$  $7^{2} = 7 \times 7 = 49$ 

The first 7 square numbers are: 1, 4, 9, 16, 25, 36, 49

#### Factors

A factor is a number that divides exactly into another number. The factors of 12 are: 1, 2, 3 ,4, 6, 12

The factors of 13 are 1 and 13



### Place value

Thousands (1000)	Hundreds (100)	lundreds Tens Ones (100) (10) (1) •		Tenths <u>1</u> 10	Hundredth <u>1</u> 100	s Thousandths <u>1</u> 1000	
10 ones 10 tens 10 hundreds	= 1 ter = 1 hur = 1 thc	ndred				dredths =	1 hundredth 1 tenth 1 unit

The placement of the digits within the number gives us the value of that digit.



#### Inverse operations

Inverse operations allow you to undo a sum.

Operator	Inverse				
	Operation				
+	-				
-	+				
÷	×				
×	÷				

We use inverse operations when we work with function machines.



### Fractions



#### Equivalent fractions

All the fractions below represent the same proportion. Therefore they are called equivalent fractions.







$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \dots$$

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \dots$$

$$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20} = \dots$$

$$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20} = \dots$$

### Decimals

A decimal is any number that contains a decimal point. The following are examples of decimals.



#### Changing decimals and fractions into percentages

To change a decimal or fraction to a percentage you have to multiply with 100%.

 $0.75 \times 100\% = 75\%$  $\frac{13}{20} \times \frac{5100}{100}\% = 65\%$ 

To change a fraction into a decimal you have to divide the numerator with the denominator.

It is also possible to change a fraction into a percentage like this:

 $\frac{2}{3}$  = 2 ÷ 3 = 0.6666 . . . = 0.67 ( to 2 decimal places ) 3 then 0.67 × 100% = 67%

Therefore 2/3 = 67% (to the nearest one part of a hundred)

#### Useful fractions, decimals and percentages

Fraction	Decimal	Percentage
1	1.0	100%
<sup>1</sup> / <sub>2</sub>	0.5	50%
<sup>1</sup> / <sub>3</sub>	0.33	33%
1/4	0.25	25%
<sup>3</sup> / <sub>4</sub>	0.75	75%
<sup>1</sup> / <sub>10</sub>	0.1	10%
$^{2}/_{10}$ ( = $^{1}/_{5}$ )	0.2	20%
<sup>3</sup> / <sub>10</sub>	0.3	30%

#### Ratio

Ratio is used to make a comparison between two things.

#### Example



In this pattern we can see that there are 3 happy faces to every sad face. We use the symbol : to represent to in the above statement, therefore we write the ratio like this:

> Happy : Sad 3 : 1

Ratio is used in a number of situations:

- In a cooking recipe
- In building when mixing concrete
- It is used in the scale of maps
   e.g. if a scale of 1 : 100 000 is used,
   it means that 1 cm on the map represents
   100 000 cm in reality which is 1 km.





### Integers / Directed numbers

The negative sign (-) tells us the number is below zero e.g. -4. The number line is useful when working with negative numbers. Below is a part of the number line.

Negative direction							$\leftarrow$	$\leftarrow$ $\rightarrow$ Positi			ositiv	e dire	ectior	The numbers		
-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	on the right
																are greater

than the numbers on the left e.g. 5 is greater than 2 and 2 is greater than -3. Note that -3 is greater than -8.

#### Adding and subtracting integers

The Number line game can be used to add and subtract negative numbers:

#### Rules:

Start at zero facing the positive direction.

Ignore any + signs.

The - sign means "make half a turn".

When you see a number, step the value of the number in the direction you are facing. After stepping, face the positive direction before continuing with the sum.

Your position at the end will be the answer.

#### **Example:** - 3 - 4 + 6

Sum	-8	-7	-6	-5	-4	-3	-2	-1	0	1	Method
									$\rightarrow$		Start at zero.
-									←		Make half a turn.
3						-					Step 3.
						$\rightarrow$					Face the positive.
-						-					Make half a turn.
4	1	-									Step 4.
		$\rightarrow$									Face the positive.
+		$\rightarrow$									Ignore the +.
6								$\rightarrow$			Step 6.
								$\odot$			The answer is -1.

#### **Example:** 2 + - 8 - - 9



- Start at zero facing the positive direction.
- Step 2 and face the positive direction.
- Ignore the + , make half a turn, step 8 and face the positive direction.
- Make half a turn, make half a turn, step 9 and note your position. The answer is **3** :

#### Multiplying and dividing integers

We multiply and divide directed numbers in the usual way while remembering these very important rules:

Two signs the same, a positive answer.

Two different signs, a negative answer.



Remember, if there is no sign before the number, it is positive.

#### Examples:

5	×	-7	=	-35	(different signs give a negative answer)
-4	×	-8	=	32	(two signs the same give a positive answer)
48	÷	-6	=	-8	(different signs give a negative answer)
-120	÷	-10	=	12	(two signs the same give a positive answer)

### Coordinates

We use coordinates to describe location.



The coordinates of the points are:

There is a special name for the point (0,0) which is the origin.

The first number (x-coordinate) represents the distance across from the origin. The second number (y-coordinate) represents the distance going up or down.

**Example** : The point (1,2) is one across and two up from the origin.

#### Inequalities

We us the = sign to show that two sums are **equal**. If one sum is greater than or less than the other we use inequalities:

	< less than		> more than	
	< less than or e	qual to	$\geq$ more than or	r equal to
Examples :				
	5 < 8	43 > 6	<i>x</i> < 8	$y \ge 17$

### Names of two dimensional shapes

A polygon is a closed shape made up of straight lines. A regular polygon has equal sides and equal angles.



### **3D** shapes

3D means three dimensions - 3D shapes have length, width and height.

Shape	Name	Faces	Edges	Vertices (corners)
Triangular Pyramid		4	6	4
	Cube	6	12	8
	Rectangular Prism	6	12	8
	Octahedron	8	12	6
	Square pyramid	5	8	5
	Triangular prism	5	9	6

#### Euler's formula:

Number of faces - Number of edges + Number of vertices = 2

#### The circle



### Circumference of a circle

The circumference of a circle is the distance around the circle.

Circumference =  $\pi \times \text{diameter}$ Circumference =  $\pi \text{d}$ 

Since the diameter is twice the length of the radius, we can also write

Circumference =  $\pi \times 2 \times radius$ Circumference =  $2\pi r$ 

## π (pi)

 $\pi$  is a Greek letter which represents 3.1415926535897932384..... (a decimal that carries on for ever without repetition)

We round  $\pi$  to 3.14 in order to make calculations or we use the  $\pi$  button on the calculator.

#### Perimeter

Perimeter is the distance around the outside of a shape. We measure the perimeter in millimeters (mm), centimeters (cm), meters (m), etc.



This shape has been drawn on a 1cm grid. Starting on the orange circle and moving in a clockwise direction, the distance travelled is . . .

1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 2 + 1 + 2 = 14cm

Perimeter = 14cm

## Area of 2D Shapes

The area of a shape is how much surface it covers. We measure area in square units e.g. centimeters squared  $(cm^2)$  or meters squared  $(m^2)$ .

### Areas of irregular shapes

Given an irregular shape, we estimate its area through drawing a grid and counting the squares that cover the shape.



Area =  $11 \text{ cm}^2$ .

**Remember** that this is an estimate and not the exact area.

## Area formula





Multiply the length with the width.



Add the parallel sides, multiply with the height and divide by two.

Area = 
$$\frac{(a+b)h}{2}$$





Multiply the radius with itself, then multiply with  $\pi$ .

Area =  $r \times r \times \pi = \pi r^2$ 

#### Triangle



Multiply the base with the height and divide by two.  $4ncc = \frac{b \times h}{b}$ 

Area = 
$$\frac{D \times R}{2}$$

#### Parallelogram



Multiply the base with the height.

#### Volume

Volume is the amount of space that an object contains or takes up. The object can be a solid, liquid or gas.

Volume is measured in cubic units e.g. cubic centimeters (cm<sup>3</sup>) and cubic meters (m<sup>3</sup>).



#### **Rectangular** Prism

Note that a rectangular prism has six rectangular faces.



Volume rect. prism= length x width x height

#### Prism

A prism is a 3-dimensional object that has the same shape throughout its length i.e. it has a uniform cross-section.



Volume of a prism = area of the base x length

## Metric units of length

mm	10 mm = 1 cm 1 000 mm = 1 m
cm	100 cm = 1 m   100 000 cm = 1 km
m	1 000 m = 1 km
km	a creating a
	cm m

## Imperial/Standard units of length

Inch	in or "	12 in = 1 ft
Foot	ft or '	3 ft = 1 yd
Yard	yd	1 760 yd = 1 mile
Mile		

### Metric units of mass

Milligram	mg	1 000 mg = 1 g   1 000 000 mg = 1 kg
Gram	g	1 000 g = 1 kg
Kilogram	kg	1 000 kg = 1 t
Metric ton	+	

## Imperial/Standard units of mass

Ounce	οz	16 oz = 1 lb
Pound	lb	2000 lb = 1 ton



## Metric units of volume

Milliliter ml 1000 ml = 1 l Liter l

### Imperial/Standard units of volume

Ounce	οz		
Сир	с		
Pint		pt	8 oz = 1 c
Quart	qt		
Gallon		gal	4 qt = 1 gal



## Converting between imperial and metric units

## Length

1 inch	~	2.5 cm
1 foot	~	30 cm
1 mile	~	1.6 km
5 miles	~	8 km

## Weight/Mass

1 pound	~	454 g
2.2 pounds	~	1 kg
1 ton	~	1 metric tonne

### Volume

1 gallon	~	4.5 litre
1 pint	~	0.6 litre(568 ml)
1ª pints	~	1 litre

°C°F

Temperature

· · · · · · · · · · · · · · · · · · ·		
Converting from Celsius (°C) to Fahrenheit (°F)	50 	_ <u>120</u>
Use the following formula		= 100
F = 1.8 × C + 32	30 20	80
Converting from Fahrenheit (°F) to Celsius (°C)	10	60
Use the following formula	0	<u>40</u> 32
C = (F - 32) ÷ 1.8	10	20
	20	0
Look at the thermometer:	<u>30</u>	20 40
The freezing point of water is 0°C or 32°	َتُ ال	
19	u	J

## Time

1000	years	=	1 millennium
100	years	=	1 century
10	years	=	1 decade
60	seconds	=	1 minute
60	minutes	=	1 hour
24	hours	=	1 day
7	days	=	1 week
12	months	=	1 year
52	weeks	~	1 year
365	days	~	1 year
366	days	~	1 leap year



## The Yearly Cycle

Season	Month	Days
	January	31
	February	28
	March	31
	April	30
	May	31
$\bigcirc$	June	30
$\bigcirc$	July	31
$\bigcirc$	August	31
	September	30
	October	31
	November	30
$\bigcirc$	December	31



## The 24 hour and 12 hour clock

	24 hour	12 hour		
Midnight	00:00	12.00 a.m.	Midnight	
	01:00	1:00 a.m.		
	02:00	2:00 a.m.		
	03:00	3:00 a.m.		
The 24 hour clock always	04:00	4.00 a.m.		
uses 4 digits to show the	05:00	5:00 a.m.	The 12 hour clock shows the	
time.	06:00	6:00 a.m.	time with a.m. before mid-	
The 24 hour system does	07:00	7:00 a.m.	day and p.m. after mid-day.	
not use a.m. nor p.m.	08:00	8:00 a.m.		
	09:00	9:00 a.m.		
	10:00	10:00 a.m.		
	11:00	11:00 a.m.		
Mid-day	12:00	12:00 p.m.	Mid-day	
	13:00	1:00 p.m.		
	14:00	2:00 p.m.		
	15:00	3:00 p.m.	Im m m mt	
17.57	16:00	4:00 p.m.	12 12	
11.26	17:00	5:00 p.m.	CONNECT AND	
Inter r	18:00	6:00 p.m.	9 3	
WONTH DATE DATE DATE	19:00	7:00 p.m.	WATER RESISTANT	
INFO-STATION	20:00	8:00 p.m.	<b>7 6 5</b>	
	21:00	9.00 p.m.		
	22:00	10.00 p.m.		
	23:00	 11:00 p.m.		

## Time vocabulary

02:10	Ten past two in the morning	2:10 a.m.
07:15	Quarter past seven in the morning	7:15 a.m.
15:20	Twenty past three in the afternoon	3:20 p.m.
21:30	Half past nine in the evening	9:30 p.m.
14:40	Twenty to three in the afternoon	2:40 p.m.
21:45	Quarter to ten at night	9:45 p.m.

### Bearings

A bearing describes direction. A compass is used to find and follow a bearing.





The bearing is an angle measured clockwise from the North.

Bearings are always written using three figures e.g. if the angle from the North is 5°, we write 005°.

# Data

We collect data in order to highlight information to be interpreted.

There are two types of data:

Discrete data	Continuous data
Things that are not measured:	Things that are measured:
• Colors	<ul> <li>Pupil height</li> </ul>
<ul> <li>Days of the week</li> </ul>	<ul> <li>Volume of a bottle</li> </ul>
<ul> <li>Favorite drink</li> </ul>	<ul> <li>Mass of a chocolate bar</li> </ul>
<ul> <li>Number of boys in a family</li> </ul>	<ul> <li>Time to complete a test</li> </ul>
• Shoe size	• Area of a television screen

## Discrete data

## Collecting and recording

We can record data in a list

e.g. here are the numbers of pets owned by pupils in form 9C:

1,2,1,1,2,3,2,1,2,1,1,2,4,2,1,5,2,3,1,1,4,10,3,2,5,1

A frequency table is more structured and helps with processing the information.

Number of pets	Tally	Frequency
1	₩T₩T	10
2	J#f	8
3		3
4	I	2
5	I	2
6		0
7		0
8		0
9		0
10		1

## Displaying

In order to communicate information, we use statistical diagrams. Here are some examples:

## Pictogram

A pictogram uses symbols to represent frequency. We include a key to show the value of each symbol.

The diagram below shows the number of pets owned by pupils in 9C.



🔌 Represents two pupils.

## Bar chart

The height of each bar represents the frequency. All bars must be the same width and have a constant space between them. Notice that the scale of the frequency is constant and starts from 0 every time. Remember to label the axes and give the chart a sensible title.



Number of animals

## Vertical line graph

A vertical line graph is very similar to a bar chart except that each category has a line instead of a bar. Notice that the category labels are directly below each line.



Pets owned by pupils of 9C

Pie chart/ Circle Graph

The complete circle represents the total frequency. The angles for each sector are calculated as follows:

Here is the data for the types of pets owned by 9C

Type of pet	Frequency	Ar	igle	of th	e s	ector	Divide 360° by the total
Cats	13	13	х	10°	=	130°	of the frequency:
Dogs	11	11	x	10°	=	110°	
Birds	5	5	x	10°	=	50°	360° ÷ <mark>36</mark> = 10°
Fish	7	7	х	10°	=	70°	Therefore 10°
Total	36					360°	represents one animal

Remember to check that the angles of the sectors add up to 360°.

## Types of pet owned by 9C



## Continuous data

### Displaying

With graphs representing continuous data, we can draw lines to show the relationship between two variables. Here are some examples:

#### Line graph

The temperature of water was measured every minute as it was heated and left to cool. A cross shows the temperature of the water at a specific time. Through connecting the crosses with a curve we see the relationship between temperature and time.



#### Heating and cooling water

The line enables us to estimate the temperature of the water at times other than those plotted e.g. at  $6\frac{1}{2}$  minutes the temperature was approximately 40 °C.

#### **Conversion graph**

We use a conversion graph for two variables which have a linear relationship. We draw it in the same way as the above graph but the points are connected with a straight line.



#### Converting between miles and km

From the graph, we see that 8 km is approximately 5 miles.

### Scatter Plot

We plot points on the scatter plot in the same way as for the line graph. We do not join the points but look for a correlation between the two sets of data.



If there is a correlation, we can draw a line of best fit on the diagram and use it to estimate the value of one variable given the other.

The following scatter graph shows a positive correlation between the weights and heights of 12 pupils.



Comparing the height and weight of 12 pupils

The line of best fit estimates the relationship between the two variables. Notice that the line follows the trend of the points.

There are approximately the same number of points above and below the line.

We estimate that a pupil 155 cm tall has a weight of 60 kg.

#### Important things to remember when drawing graphs

- Title and label axes
- Sensible scales
- Careful and neat drawing with a pencil

## Average

The average is a measure of the middle of a set of data. We use the following types of average:

- Mean We add the values in a set of data, and then divide by the number of values in the set.
- Median Place the data in order starting with the smallest then find the number in the middle. This is the median. If you have two middle numbers then find the number that's halfway between the two.
- Mode This is the value that appears most often.

## Spread

The spread is a measure of how close together are the items of data. We use the following to measure spread:

Range - The range of a set of data is the difference between the highest and the lowest value.

## Example

Find the mean, median, mode, and range of the following numbers:

4,3	, 2 , 0 , 1 , 3 , 1 , 1	, 4	5
Mean	4 + 3 + 2 + 0 + 1 + 3 + 1 + 1 + 4 + 5		= 2.4
Mean	10		
Median	0 , 1 , 1 , 1 , <mark>2</mark> , <mark>3</mark> , 3 , 4 , 4 , 5	2+3 2	= 2·5
Mode	0, 1, 1, 1, 2, 3, 3, 4, 4, 5		= 1
Range	<mark>0</mark> , 1 , 1 , 1 , 2 , 3 , 3 , 4 , 4 , <mark>5</mark>	5 - 0	= 5

# Vocabulary

Acceleration	
Acute angle	
Add	
Angle	
Approximation	
Area	
Average	
Axis	
Balance	
Bearing	
Bills	
Bisect/Halve	
Boundary	
Calculator	
Capacity	
Cash	
Circle	
Circumference	
Clockwise	
Column	
Compass (drawing circles)	
Compass (points North)	
Cone	
Co-ordinates	
Corresponding	
Counter-clockwise	
Cross-section	
Cube	
Curve	
Cylinder	
Cheapest	
Decimal	

Density
Deposit
Depth
Diagonal
Diameter
Dice
Digit
Dimension
Discount
Drawn to scale
East
Edge
Enlarge
Equal/Unequal
Equivalent
Estimate
Even number
Extend
Factor
Fraction
Frequency
Gradient (slope)
Height
Horizontal
Index
Interest (rate)
Intersection
Interval
Invest
Irregular
Layer/Tier
Length
Length
Loan
Loan

Mass
Maximum
Mean
Measure
Median
Minimum
Mode
Multiple
Net
North
Obtuse angle
Octagon
Odd number
Parallel
Percent
Perimeter
Perpendicular
Power
Pressure
Prime number
Prism
Probability
Profit
Pyramid
Quadrilateral
Radius
Range
Rate of exchange
Ratio
Rectangle
Reduce/Decrease
Reflection
Reflex angle
Remainder
Right angle
Round off

Row
Salary (income)
Save
Scale
Solution
South
Space
Speed
Sphere
Square
Square number
Square Root
Substitute
Symmetry
Total
Triangle
Triangular number
Unknown
Unlikely
Velocity
Vertex
Vertical
Voi moai
Volume
Volume